Young Mathematicians Conference (YMC) 2026 Abstract Book

August 4, 2026

Contents

1 Plenary Talks

1.1 Mathematics of generative diffusion models

Maria Han Veiga (The Ohio State University)

Abstract

Diffusion probabilistic models have become the state-of-the-art tool in generative methods, used to generate high-resolution samples from very high-dimension distributions (e.g. images). Although very effective, they suffer some drawbacks: 1) as opposed to variational encoders, the dimension of the problem remains high during the generation process and 2) they can be prone to memorization of the training dataset.

In this talk, we first provide an introduction to generative modeling, with a focus on diffusion models from the point of view of stochastic PDEs. Then, we introduce a kernel-smoothed empirical score and study the bias-variance of this estimator. We find improved bounds on the KL-divergence between a true measure and an approximate measure generated by using the smoothed empirical score. This score estimator leads to less memorization and better generalization. We demonstrate these findings on synthetic and real datasets, combining diffusion models with variational encoders to reduce the dimensionality of the problem.

1.2 Geometric and arithmetic aspects of hyperbolic manifolds

Jean-François Lafont (The Ohio State University)

Abstract

Hyperbolic manifolds feature a rich interface between geometry, dynamics, and topology. I will start with a brief introduction to hyperbolic manifolds, focusing on the construction of closed hyperbolic manifolds, both arithmetic and non-arithmetic. I will then discuss some recent work on the presence of totally geodesic submanifolds, and how the presence of these is related to the arithmetic structure of the hyperbolic manifold.

1.3 Shape Matters: From the Riemann Mapping Theorem to the Bergman Projection in Several Complex Variables

Yunus Zeytuncu (University of Michigan-Dearborn)

Abstract

In one complex variable, the Riemann Mapping Theorem offers beautiful geometric flexibility. The unit disc and unit square are equivalent. In Several Complex Variables, this simplicity breaks down. The unit ball and polydisc are not holomorphically equivalent in higher dimensions. This geometric rigidity raises a natural question: what are the biholomorphic equivalence classes? When two nice domains are biholomorphically equivalent, how does the mapping behave at their boundaries? Can some boundary invariants answer the equivalence question? We will explore this via Fefferman's 1974 breakthrough, which proved that any biholomorphic map between strictly pseudoconvex domains with smooth boundaries extends smoothly to the boundary. Because solving such geometric problems requires powerful analytic tools, the talk concludes with an accessible overview of the Bergman projection. We will discuss how its analytic properties are intimately tied to the specific geometry of the underlying domain.

2 Student Participants Talks

2.1 A Spline-Based Time Series Extrapolation Model for Stock Price Forecasting (ID = 6)

Jaden Nestor-Fox (Bethany College) foxbuziness2021@gmail.com
Bethany College [Mentor: Adam C. Fletcher]

Abstract

Mathematical forecasting models have long been a topic of interest in the field of quantitative finance (Tsay, 2010; Hull, 2018). However, existing models have only relied on common statistical methods to predict markets. This paper proposes a new method of forecasting using mathematical methods on price data, such as concavity and slope, cubic spline interpolation, forward extrapolation, and Kalman smoothing. This method combines these ideas with traditional technical analysis techniques to create a smooth predictive curve that will help estimate trend behavior and turning points. Multiple back tests show that the model improves common short-term predictions relative to other prediction techniques. As a successful model, this research is original in the financial world and can be built upon to develop deeper, more intelligent, forecasting models.

2.2 Keeping it Real(izable): Realizability of Saturated Transfer Systems for Cyclic Groups (ID = 8)

Alena Gurevich (University of Michigan) alenadg@umich.edu
Edward Y. Lee (University of Michigan) lyselee@umich.edu
University of Michigan [Mentor: Bar Roytman]

Abstract

Equivariant homotopy theory has families of generalizations of commutative monoids with G-actions, where G is a finite group. These generalizations arise from G-operads and differ in which symmetric multiplication operations they admit. A G-transfer system is a combinatorial object that encodes these generalizations. Complex linear isometries operads can be used to turn a collection of irreducible G-representations into a transfer system. However, we are interested in the inverse direction. For which G-transfer systems does such a collection, a "G-universe," exist? Which transfer systems are "realizable"? When G is cyclic, this question reduces to an incidence geometry problem in the subgroup lattice of G. We are especially interested in cyclic groups whose order has at most three distinct prime-power factors. To tackle large families of transfer systems for these groups, our method for realizability involves strategically omitting cosets from the complete set of representations.

2.3 A Recurrence-Based Summation Method for Ultra-Rapidly Diverging Series and Renormalon-Type Expansions (ID = 10)

Ishan Joshi ishan.joshi@berkeley.edu
The Ohio State University [Mentor: Ovidiu Costin]

Abstract

Classical summation methods are often organized around fixed growth regimes: standard Borel summation treats Gevrey-1 series, while higher-order Gevrey behavior typically requires changing the kernel. We introduce a recurrence-based method, called C-summation, whose primary input is a first-order inhomogeneous recurrence rather than a prescribed growth class. Since the recurrence solution is determined only up to a homogeneous term, we pass to a normalized tail, where this ambiguity becomes an additive constant, and define the value by extracting a finite part. The selector is constructed through a Bromwich transform of the normalized tail differences. Under a precise $\mathcal{M}$-admissibility hypothesis, we prove independence from the chosen recurrence solution and from admissible presentations of the same

formal series, as well as regularity, homogeneity, and shift-stability. The method agrees with Borel-Laplace summation on a Borel-summable class, but also applies beyond the classical Borel setting: it sums examples growing faster than any power of the factorial, such as terms of order $e^{n^2/2}$, and assigns a principal-value Écalle-Borel value to a non-Borel-summable renormalon-type series.

2.4 Optimizing Stellar Subdivisions for Toric Resolution of Singularities (ID = 11)

Garrett C. Rhoads (Hamilton College) grhoads@hamilton.edu
Felix Santiago (Princeton University) fsantiago@princeton.edu
University of Michigan-Dearborn [Mentor: Yunus Zeytuncu]

Abstract

Resolution of singularities is a central problem in algebraic geometry, classically studied through Hironaka's work and its many later developments. Toric singularities admit a combinatorial description in terms of rational polyhedral cones. A cone is smooth precisely when its primitive ray generators extend to a lattice basis, and resolving a toric singularity amounts to subdividing singular cones into unimodular ones. This project studies the optimization problem underlying this process: How can one choose lattice rays so as to minimize the number of stellar subdivisions needed to obtain a smooth subdivision? There is an optimal choice for the 2D case given by Hirzebruch-Jung continued fractions, but beyond that not much is known. We formulate cone smoothing as a discrete-geometric search problem on lattice cones. The multiplicity of a cone, given by the lattice index of its ray generators, serves as a measure of singularity. Candidate subdivisions are analyzed using fundamental parallelepipeds, Hilbert basis elements, quotient-lattice structure, and unimodular equivalence of cones. In particular, we establish a canonical form for generating and classifying three-dimensional simplicial cones up to unimodular equivalence, allowing redundant instances to be identified and removed from the search space. We have obtained results for several structured classes of cones, including explicit optimal subdivision behavior in special families and computational evidence for broader patterns in ray selection. In particular, in the 3D case, we consolidated results from the literature to prove that selecting rays from a smaller set is sufficient for optimal subdivision. To expand and generalize these results, we develop a learning-based framework that combines graph neural networks with reinforcement learning to study cone smoothing as a sequential decision problem. In this framework, a fan is represented as a heterogeneous graph whose nodes encode cones and candidate lattice rays, while the learning agent predicts which stellar subdivision is most likely to reduce the remaining smoothing complexity. With this novel graph-based reinforcement learning approach, we aim to learn ray-selection policies that produce efficient and optimal smoothings of n-dimensional cones.

2.5 Harmonic Measure on the Basilica Julia Set: A Bridge between the Riemann Mapping Theorem and Symbolic Dynamics (ID = 12)

Chenzixi Zhao (Purdue University) zhao1452@purdue.edu
Purdue University [Mentor: Shuyi Weng]

Abstract

Julia sets are fractals arising from the iteration of rational maps in complex dynamics. For the Basilica Julia set, symbolic dynamics can be encoded by geometric addresses consisting of an initial label from $\{t, l, b, r\}$ followed by an iterative replacement rule using the symbols $\{1, 2, 3\}$. We study the distribution of these addresses by determining the probability that a randomly chosen point has a prescribed symbol at the n-th position for $n \geq 2$. By pulling back harmonic measure from the unit circle via the Riemann map, we obtain a probability distribution of $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$, visualized through external rays. Finally, we construct a fractal function that conjugates this measure to the uniform distribution $\frac{1}{3}$, $\frac{1}{3}$, and $\frac{1}{3}$, providing a new probabilistic interpretation of symbolic addresses on the Basilica.

2.6 Gelfand-type problem for turbulent jets: An L^∞ bound on extremal solutions (ID = 13)

Alex Czemerinski alexczeme@gmail.com
Philip Yao (Brown University) philip_yao@brown.edu
Kent State University [Mentor: Fedor Nazarov, Peter Gordon]

Abstract

We consider a Gelfand-type problem for turbulent jets, which amounts to a semi-linear elliptic equation $-\Delta u - \alpha r\varphi(r)\frac{\partial}{\partial r}u = \lambda\psi(r)f(u)$ in the unit disk with zero Dirichlet boundary conditions. Here, f is positive, increasing, convex, satisfying $\int_0^\infty \frac{ds}{f(s)} < \infty$, φ and ψ are non-increasing Lipschitz functions on $[0,1)$ such that $\varphi(0) = \psi(0) = 1$ and $\frac{\psi(x)}{\varphi(x)}$ does not blow up too fast as $x \to 1^-$, and $\lambda > 0$, $\alpha > 0$ are parameters characterizing the strength of the reaction and the flow rate, respectively. In [GMN], a sharp bound on the critical $\lambda^*(\alpha)$, the largest λ for which a positive solution exists for a given α, has been obtained. It has also been shown there that $u^*_\alpha(0) \to \infty$ as $\alpha \to \infty$ and an asymptotic bound on $u^*_\alpha(0)$ has been found under additional assumptions on the non-linearity. We show that the same bound holds without any such additional assumptions on f. Namely, we prove that there exist $c, C \in (0,\infty)$ such that, for sufficiently large α, one has $u^*_\alpha(0) \le cA$, where A solves $f'(A) = C\log\alpha$.

References [GMN] P. V. Gordon, V. Moroz, F. Nazarov, Gelfand-type problem for turbulent jets, J. Differential Equations 269 (2020), 5959-5996

2.7 Braided Monoidal Category of Superselection Sectors via Connes Fusion (ID = 14)

Charlton Li (The Ohio State University) li.12635@osu.edu
The Ohio State University [Mentor: David Penneys]

Abstract

Given a net of operator algebras on a poset $\mathcal{P}$, one can study its representation theory, the category SSS of superselection sectors. Using localized endomorphisms, it was shown in [arXiv: 2410.21454] that SSS is canonically a braided monoidal category (i.e., comes equipped with a tensor product and commutativity constraint) whenever $\mathcal{P}$ satisfies certain geometric axioms. We show that an equivalent braided monoidal structure on SSS can be constructed using the Connes fusion tensor product of Hilbert space bimodules over von Neumann algebras. We aim to apply this new approach to study nets of algebras indexed by cones in $\mathbb{R}^2$ arising from unitary braided fusion categories.

2.8 A Numerical and Theoretical Analysis of the Porous Medium Equation with Reaction and its Limit (ID = 15)

Joshua S. Nauman (Michigan State University) naumanj3@msu.edu
Michigan State University [Mentor: Olga Turanova, Christian Parkinson]

Abstract

We consider a porous medium partial differential equation (PDE) with parameter $m > 1$ that models tissue growth of a tumor,

$$u_t - (u^m)_{xx} = u\left(\mu(x) - p\right),$$

where $\mu : \mathbb{R} \to \mathbb{R}$ represents the favorability of the tissue's environment at a point x and the pressure p is given in terms of the density u by $p = \frac{m}{m-1}u^{m-1}$. Our goal is to find criteria on μ that determine whether the tumor grows or shrinks. This issue is well-understood for linear diffusion equations, that is, if $(u^m)_{xx}$ is replaced by u_{xx}, but very little is known in the porous medium case.

We approach this problem from both numerical and analytical perspectives. It is known that in the limit as $m \to \infty$, the PDE becomes a free-boundary problem, with $\overline{p} := \lim_{m\to\infty} p$ satisfying:

$$\begin{cases} -\overline{p}_{xx} = \mu(x) - \overline{p} & \text{on } (0, r(t)) = \{x \in (0,\infty) \mid \overline{p}(x,t) > 0\}, \\ \dot{r}(t) = -\overline{p}_x(r(t),t), & \\ r(0) = r_0 > 0, \quad \overline{p}(0,t) = 1, \quad \overline{p}(r(t),t) = 0. & \end{cases}$$

We prove the existence, uniqueness, and non-negativity of solutions to this system. We study how various conditions on μ affect the asymptotic behavior of $r(t)$.

To obtain our numerical results, we model the PDE using an implicit finite-difference method and use our empirical findings to develop conjectures about conditions on μ that guarantee either tumor growth or extinction.

2.9 A Spectral Characterization of Perfect State Transfer in Non-Hermitian Tridiagonal Matrices (ID = 16)

Adrian deCola (Imperial College London) adriandecola@gmail.com
Adam Deinhardt (Columbia University) amd2410@columbia.edu
Andrew Rittenberg (University of Hartford) rittenber@hartford.edu
Emily Shteynberg (New York University) es5696@nyu.edu
University of Connecticut [Mentor: Maxim Derevyagin and Max Meynig]

Abstract

In quantum mechanics, the states of quantum particles are described using probability. Perfect state transfer (PST) occurs when a quantum state localized at one site evolves to a target site with probability one. Equivalently, PST from the first site to the N-th site occurs when there exist some time $t > 0$ and $\varphi \in \mathbb{R}$ such that

$$e^{it\mathbf{H}}\mathbf{e}_1 = e^{i\varphi}\mathbf{e}_N,$$

where $\mathbf{H}$ is a Hermitian tridiagonal matrix, and $\mathbf{e}_1$ and $\mathbf{e}_N$ are standard basis vectors corresponding to the first and N-th sites, respectively. A. Kay gave a spectral characterization of Hermitian tridiagonal matrices $\mathbf{H}$ that realize PST. We extend this result to a class of non-Hermitian tridiagonal matrices that arise in models recently studied by physicists (X. Z. Zhang et al., M. Znojil). In particular, we show that these matrices achieve PST exactly when $\mathbf{H}$ is symmetric across the antidiagonal, and the spacings between consecutive eigenvalues λ_k are given by $\lambda_{k+1} - \lambda_k = m_k \dfrac{\pi}{t}$ for some $m_k \in \mathbb{Z}$ for $k = 1, 2, \ldots, N-1$. We then provide an algorithm for constructing such matrices from some sets of eigenvalues that satisfy the condition.

2.10 Inclusion-Minimal Lattice Polygons of Fixed Lattice Size (ID = 18)

Alexander Coutras (University of Maryland - College Park) acoutras@icloud.com
Kent State University [Mentor: Jenya Soprunova]

Abstract

The lattice size of a lattice polygon was introduced and studied by Schicho, and by Castryck and Cools in relation to the problem of bounding the total degree of the defining equation of an algebraic curve. Given a lattice polygon $P \subset \mathbb{R}^2$, its lattice size with respect to the standard simplex Δ is the smallest integer ℓ such that P can be mapped inside the ℓ-multiple of Δ via an affine unimodular map. Given a positive integer ℓ, we provide a complete classification of lattice polygons of lattice size ℓ that are minimal with respect to inclusion. We use this classification to establish a sharp upper bound on the lattice size of a lattice polygon P in terms of its area. Our main tool is a result by Harrison and Soprunova that states that a reduced basis (that is, a basis of directions of the shortest width of P) computes the lattice size of P.

2.11 Edge-Informed PSF Estimation (ID = 19)

Toby Shu (Georgetown University) tobyshu0@gmail.com
Nicholas P. Yamine (University of Michigan-Dearborn) npyamine@umich.edu
University of Michigan-Dearborn [Mentor: Aditya Viswanathan]

Abstract

Imaging devices capture the real world with a blur. This blur is modeled by convolving the true image with a blurring function, also known as a point spread function (PSF). Often, PSFs are estimated by capturing an idealized diagnostic point source to see how an individual point is blurred. We instead estimate PSFs in the more realistic setting where the true image is piecewise-smooth and no diagnostic point source is available. Through the use of a convolutional edge detection kernel, each edge is transformed into a diagnostic point. We present approximation theoretic L^2 error bounds for estimations of Laplace and Cauchy PSFs. We also show that, for certain PSF families, the shape parameter is a generalized eigenvalue of a pair of Fourier domain Hankel matrices. We then combine these classical PSF estimation techniques with contemporary machine learning approaches for classifying unknown families of PSFs and estimating their parameters.

2.12 Spectra of the Weighted Laplacians on the Schreier Graphs of the Basilica Group (ID = 21)

Carter V. Oxford (Angelo State University) cartervoxford@gmail.com
Tanvi K. Skanda (Hendrix College) kirantt@hendrix.edu
Delaney L. Thomas (University of Findlay) delaneythomas117@gmail.com
University of Connecticut [Mentor: Luke Rogers]

Abstract

Grigorchuk and Zuk introduce the Basilica group, a self-similar group generated by the 3-state automaton in [1]. Combined with the results of Bartholdi and Virag in [2], the Basilica group is established as the first example of an amenable but not subexponentially amenable group, and has become a foundational example in the theory of self-similar groups. The self-similar structure of the Basilica group's Laplacian spectrum was already recognized in [1], but difficult to analyze; whereas [4] offers an alternative approach by conducting analysis on a macroscopic unweighted decomposition as compared to the microscopic weighted structure discussed in [3] by Grigorchuk and Zuk. Generalizing the work of Brzoska et al. in [4], Bannon in [5] and Ambrose et al. in [6], we analyze the Laplacian spectrum of the weighted Schreier graphs of the Basilica group. Investigating the weighted Laplacians of a decomposition of the Schreier graphs, we find that the invariant two-dimensional dynamical system of the spectra of these Laplacians have similar structure to the results of [4], expanding Brzoska et al's work on the matter to the weighted graphs originally investigated by Grigorchuk and Zuk. We develop a complete recursive structure to the characteristic polynomials of the conformally invariant Laplacian for each Schreier graph decomposition. Additionally, we can factor the characteristic polynomials according to the homology of the weighted graphs. We aim to show that the spectrum of the Laplacian for the one-parameter family of weighted graphs is a Cantor set.

References: [1] Grigorchuk, Rostislav I., and Andrzej Zuk. “On a torsion-free weakly branch group defined by a three state automaton.” International Journal of Algebra and Computation 12, no. 1-2 (2002): 223-246. [2] Bartholdi, Laurent, and Bálint Virág. “Amenability via random walks.” (2005): 39-56. [3] Grigorchuk, Rostislav I., and Andrzej Zuk. “Spectral properties of a torsion-free weakly branch group defined by a three state automaton.” Contemporary Math. 298 (2002): 57-82. [4] Brzoska, Antoni, Courtney George, Samantha Jarvis, Luke G. Rogers, Alexander Teplyaev. “Spectral properties of graphs associated to the Basilica group.” arXiv:1908.10505 (2019). [5] Bannon, Anne D. ”Spectral Decimation on the Schreier Graphs of the Basilica Group: A Thesis in Fractal Analysis.” Scripps Senior Theses (2026). [6] Ambrose, Kyle, Noah Dunham, Michael Morris, Luke G. Rogers, Alexander Teplyaev. ”Spectral properties of the Schreier graphs of the basilica group” arXiv:2606.07430 (2026).

2.13 A Variant of a Problem of Erdős and Rényi: Bounds on the Asymmetric Index of a Graph (ID = 22)

Robert Hilbert(Rochester Institute of Technology) rhh8366@rit.edu
Jackson Welde (Rochester Institute of Technology) jpw9845@rit.edu
Rochester Institute of Technology [Mentor: Darren Narayan]

Abstract

We identify an error in a construction given by Erdős and Rényi of an asymmetric graph that can be made non-asymmetric by removing an edge, but not by adding one. Then we provide a construction of an infinite family of graphs that realizes this property. A graph G is *asymmetric* if its automorphism group is trivial. Asymmetric graphs were introduced by Erdős and Rényi (1963). Castano et al. (2025) defined the *asymmetric index* of a graph G, denoted $ai(G)$, to be the minimum number r edges removed and s edges added so that the resulting graph G is asymmetric. We investigate upper and lower bounds on the asymmetric index of complete multipartite graphs and the disjoint union of two cycles. We also improve the best known upper bound for the maximum asymmetric index of a graph.

2.14 Perfect State Transfer in Cycles (ID = 23)

Abigail K. Clasemann (Gustavus Adolphus College) aclasemann@gustavus.edu
Gillian E. Lyons (University of Arizona) gelyons04@gmail.com
Preston Q. Vo (University of California Berkeley) pvo2020work@gmail.com
University of Connecticut [Mentor: Maksym Derevyagin, Max Meynig]

Abstract

In an interacting chain of particles, which we call a spin chain, perfect state transfer (PST) occurs if there is a complete transfer of a quantum state at some time from one end of the spin chain to the other – an important concept in the field of quantum information theory. A 1D system with nearest neighbor interactions in the spin chain is represented using a weighted adjacency matrix H, which is tridiagonal. PST occurs from the first site to the Nth site at time t with respect to some real constant ϕ if the following equation holds:

$$e^{itH}\vec{e_1} = e^{i\phi}\vec{e_N},$$

where $\vec{e_1}$ and $\vec{e_N}$ are standard basis vectors. In 2010, Kay gave a spectral characterization of the class of such matrices H that achieve PST. In the paper published in 2020 by Derevyagin, Dunne, Mograby, and Teplyaev, an algorithm that lifts a 1D spin chain onto various graphs was proposed. In particular, a Hamiltonian on a $(2n-2)$-particle cycle with PST going from the left-most to the right-most particle can be generated from an n-particle spin chain with PST. The resulting Hamiltonian is a weighted adjacency matrix of a cycle, which is not tridiagonal. This algorithm raises the natural question whether there is a one-to-one correspondence between 1D chains and cycles with PST. In our work, we construct examples of Hamiltonians that realize PST on a cycle that cannot be obtained using the algorithm, showing that the set of all Hamiltonians with PST on cycles is larger than the set of all Hamiltonians with PST on 1D chains. Moreover, we prove partial results for the characterization of Hamiltonians that realize PST on cycles.

2.15 Covering the Numerical Range of Operators with Berezin Ranges on Subspaces of the Hardy Space (ID = 24)

Gregory Yeghiyan gyeghiya@umich.edu
University of Michigan-Dearborn [Mentor: John Clifford]

Abstract

The numerical range of a bounded operator T on a Hilbert space $\mathcal{H}$ is defined as

$$W(T) = \{\langle Tx, x\rangle \mid \|x\| = 1,\ x \in \mathcal{H}\}\,.$$

When $\mathcal{H}$ is a reproducing kernel Hilbert space (RKHS) with normalized reproducing kernels k_p, $p \in X$, the Berezin range of T, which is a subset of the numerical range of T is given by

$$Ber(T) = \{\langle Tk_p, k_p\rangle \mid p \in X\}\,.$$

The weighted shift operator on the space of polynomials of degree 2 or less, $P_2(\mathbb{D}) \subset H^2(\mathbb{D})$, is defined by $T(1) = 0,\ T(z) = a,\ T(z^2) = bz$, where $a, b \in \mathbb{C}$. By considering the cone in $\mathbb{C}^3$ formed by the coefficients in the collection of all possible unit vectors x_θ that map $\langle Tx_\theta, x_\theta\rangle$ to $\partial W(T)$ and the number of intersubsections with the collection of vectors formed by Uk_p with fixed unitary U, we show that only at most 4 intersubsections occur per unitary. Consequently, $W(T)$ cannot be covered by any countable union of Berezin ranges of its unitary conjugates; instead, an uncountable union is required. In $P_1(\mathbb{D})$, it is known that three unitary conjugates will suffice. Rank one operators can be expressed in the form $Tf = \langle f, g\rangle h$, where $\|h\| = 1$. By showing that any unit vectors mapping to $\partial W(T)$ must belong to the two-dimensional subspace span$\{g, h\}$ and exploiting linear independence of reproducing kernels on our chosen spaces, we show that, for such operators T on $H^2(\mathbb{D})$ or $P_n(\mathbb{D})$ with $n \geq 2$ and $g \neq ch$, it takes the union of uncountably many Berezin ranges of unitary conjugates of T to cover $W(T)$. In contrast, if $g = ch$, then it is known that two unitary conjugates will suffice.

2.16 Using Virtual Persistence Diagrams to Improve Protein Structure Prediction Models (ID = 25)

Ahmed B. Guler (University of Georgia) abg13099@uga.edu
Henry M. Williamson (Utah Tech University) henwill8@yahoo.com
Georgia State University [Mentor: Mehmet Aktas]

Abstract

Transformer architectures designed for protein structure prediction such as ESMFold by Meta have been very successful. These architectures take an amino acid sequence and output a folded protein. They are trained on loss terms that focus on Euclidean distances which do not capture the entire geometry of a folded protein. To address this gap, we represent proteins' geometry by assigning proteins a weighted graph representaton, and using persistent homology to generate persistence diagrams of these graphs. Following developments in compuational topology, we treat these diagrams as elements of the free commutative monoid on $R^2_{\leq}$. Under certain conditions, we are able to guarantee that the Grothendieck completion of this monoid is a locally compact and abelian group, and by defining heat kernels on its Pontryagin dual we are able to construct a reproducing kernel Hilbert space into which we can naturally embed our persistence diagrams. Approximating this Hilbert space using Markov chain Monte Carlo sampling, we are able to define new loss topological terms - and by training ESMFold to minimize these losses we improve its output across two biological metrics of confidence.

2.17 On Self-Organized Criticality: Probability Theory in Solar Flares (ID = 27)

Chidiebere Nelson Okafor (Colby College) cnokaf28@colby.edu
Colby College [Mentor: Matthew Jones]

Abstract

A Self-organized Criticality (SOC) depicting system is a Dynamical Systems that arrange, or more colloquially, tune itself so that it always sits at the critical point, no matter what state one starts in, in the system's evolution. Solar flares are one powerful example of a natural system exhibiting SOC, where the intensity of events follows a power-law distribution. Our research develops a probabilistic model for

solar flare intensities using a continuous power-law distribution motivated by the need to understand the likelihood of both "typical" and extreme events.

Our primary contribution is threefold. First, we present a theorem proving that a power-law-distributed random variable has only finitely many well-defined moments. Applying this theorem to solar flares, we derive a key mathematical result: the power-law exponent α is estimated to be 1.278 ± 0.003 via the Markov-Chain Monte Carlo (MCMC) simulation we ran on the dataset of solar flare intensities drawn from: Solar Flare Catalog. Since $\alpha < 2$, following an observation from our analysis, the distribution has an infinite mean and so violates the Weak Law of Large Numbers and lacks a "typical" expectation.

By conditioning on a physical upper bound (the intensity of the Carrington Event — the most intense geomagnetic storm in recorded history), we compute a finite conditional expectation of $8.70 \times 10^{-5} W/m^2$ and a variance of $1.57 \times 10^{-7} (W/m^2)^2$, providing meaningful insights into "typical" flare behavior.

Then, we analyze the model's extreme behavior and prove two results regarding the inevitability of catastrophic "superflares." We show that the probability of at least one flare exceeding a "lethal" threshold approaches 1 as the number of flares increases. Critically, by extending the Strong Law of Large Numbers to non-negative random variables with infinite mean, we prove that the long-term average intensity of solar flares diverges to infinity almost surely, suggesting that lethal events will not only occur but will eventually represent below-average behavior.

The proven theorems include:

A. (**Finitely Many Well-Defined Moments**) Let X be a random variable which follows the power law. X has only finitely many well-defined moments.

B. (**The Extinction of Man**) With probability 1, there will be a solar "super" flare with X-ray intensity large enough to overcome Earth's radiation shielding.

C. (**It Gets Worse**) Let $X_1, X_2, \dots$ be i.i.d. random variables representing solar flare intensities, each following a power law distribution with exponent $\alpha < 2$, so that $\mathbb{E}[X_1] = +\infty$. Let $L = 45$ denote the "lethal" intensity threshold past which solar flares may be damaging to life. Then

$$\mathbb{P}\left(\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} X_i = +\infty \right) = 1,$$

and in particular, the mean of the running sample eventually and permanently exceeds L almost surely. So "lethal" events not only happen but happen often enough that eventually they will represent average behavior and eventually also below-average behavior.

2.18 Regularization Independence in the One-Dimensional Schrodinger Equation with Delta Potential (ID = 28)

Alex Czemerinski (University of Florida) alexczemerinski@ufl.edu
Jonathan L. Nunes (University of Florida) jlnunes912@gmail.com
University of Florida [Mentor: Sergei Shabanov]

Abstract

We study the regularization independence of the bound state energy and scattering coefficients in the one-dimensional Schrodinger equation with a sequence of potentials which converge to the Dirac delta-distribution potential. While there is a standard result for the delta potential in 1D, many methods of obtaining this result involve employing some regularization of the delta-function. We're interested in finding the biggest class of sequences of functions which converge to the delta dirac distribution for which the bound-state energy and scattering amplitudes can be shown to converge to the known results. We demonstrate that for regularizations satisfying some mild assumptions—boundedness, integrability, decay, normalization—the observables converge to the well-known results. We also demonstrate the existence of regularizations, which break some of our assumptions, for which the Bound state Energy does not converge to the known result.

2.19 Formal verification of the algebra of Furstenberg families in Ramsey Theory (ID = 28)

Angelina Blahodatna (University Of Massachusetts Lowell) Angelina_Blahodatna@student.uml.edu
Lauren A. Detmold (University Of Massachusetts Lowell) lauren_detmold@student.uml.edu
University Of Massachusetts Lowell [Mentor: Daniel Glasscock]

Abstract

This project introduces an algebra on Furstenberg families, organizational objects central to modern Ramsey Theory. Beyond streamlining some well-known operations, our framework underpins new generalizations of results by Bergelson, Furstenberg, and Weiss. To ensure robustness, we formally verified the algebra in the Lean proof assistant, a process which highlights the importance of exhaustive and unambiguous definitions. In this talk, we will define Furstenberg families, outline the motivating context, showcase examples of the algebra, and demonstrate the formal verification workflow.

2.20 Planar Actions of the Klein Bottle and Similar Groups (ID = 30)

Ryan E. Berman (University of Virginia) tyj7jr@virginia.edu
Carson C. Harter (University of Virginia) vsf6vt@virginia.edu
Srinivasan Kannan (University of Virginia) bff5wv@virginia.edu
University of Virginia [Mentor: Darien Jade Farnham)]

Abstract

We first present a result of F. Le Roux, which shows that for any free action of the fundamental group of the Klein Bottle, one generator is forced to be conjugate to a translation, while the other can never be. We then attempt to extend this type of result to other finitely-presented torsion-free groups whose relations are similar. The overarching goal of our work is to investigate the connection between the defining relations of a finitely presented group and the dynamics of its free actions on the plane.

2.21 Continuation of Finite-Time Collisions in the Vanishing Repulsion Limit (ID = 34)

Andrew R. Kulik (University of Wisconsin Madison) arkulik@wisc.edu
University of Wisconsin Madison [Mentor: Dallas Albritton)]

Abstract

We investigate how to continue solutions beyond a finite-time collision in two Hamiltonian systems: the Kepler and anisotropic Kepler problems. The Kepler problem is regularizable in the sense that given an ε perturbation of a collision orbit, taking $\varepsilon \to 0$ recovers a unique continuation beyond collision. The anisotropic Kepler problem on the other hand is not regularizable. We explore techniques of "regularizing" both problems by modifying the potential. For example, the modified anisotropic Kepler potential:

$$\ddot{\vec{q}} = \nabla_{\vec{q}} \frac{1}{(\mu q_1^2 + q_2^2)^{1/2}} + \kappa \frac{\vec{q}}{|\vec{q}|^{\nu+3}}\,, \quad \mu \geq 1\,,\ \nu > 0$$

with $0 < |\kappa| \ll 1$. By matched asymptotic analysis, we discover a critical threshold $\varepsilon \sim \kappa^a$ separating the behaviors – and, in particular, whether the continuations are unique – of the possible double limits as $\varepsilon, \kappa \to 0^+$. In particular, for $\kappa = 0$ we develop a description of near collision orbits by matching the canonical outer problem to a transformed inner problem using a change of variables due to McGehee. We show that it is sufficient to consider a parameterized zero-energy manifold in the inner problem on which the behavior has been explored by Devaney and others. We further corroborate our findings with numerical simulations.

2.22 On the existence of hyperelliptic curves over $\mathbb{Q}(T)$ with lower Jacobian ranks via generalized Nagao's conjecture

(ID = 35)

Joshua Rohyun Im (Texas A&M University) jri@tamu.edu
Lucas Chen *Williams College* [Mentor: Steven J. Miller)]

Abstract

An elliptic curve $E/\mathbb{Q}$ over a field $\mathbb{Q}$ is a curve $y^2 = x^3 + ax + b$, where a, $b \in \mathbb{Z}$. The set of all points on it forms a finitely generated abelian group:

$$E(\mathbb{Q}) \;\cong\; \mathbb{Z}^r \oplus \mathbb{Z}_{q_1} \oplus \cdots \oplus \mathbb{Z}_{q_t},$$

with r the rank of E. The rank is an important factor of an elliptic curve, and a lot of work has been done with it.

More generally we can study elliptic curves over $\mathbb{Q}(T)$, defined by $y^2 = x^3 + a(T)x + b(T)$, where $a(T)$, $b(T) \in \mathbb{Z}[T]$. A specialization to an integer $T = t$ gives an elliptic curve $\mathcal{X}_t$ over $\mathbb{Q}$.

The cubic polynomial in x could be generalized, giving hyperelliptic curves of genus g, by $\mathcal{X} : y^2 = f(x, T)$ where the degree of f in x is either $2g + 1$ or $2g + 2$. Since the points in hyperelliptic curves generally do not form a group, we instead consider the Jacobian variety $J_{\mathcal{X}}(\mathbb{Q}(T))$, carrying a natural group structure, hence letting us define the rank.

Let $\mathcal{X} : y^2 = f(x, T)$ be a hyperelliptic curve of genus $g \geq 1$, defined over $\mathbb{Q}(T)$. If we let $a_{\mathcal{X}_t}(p) = p + 1 - \#\mathcal{X}_t(\mathbb{F}_p)$ for a prime p, where

$$\#\mathcal{X}_t(\mathbb{F}_p) \;=\; \{(x, y) \in \mathbb{F}_p^2 : y^2 \;\equiv\; f(x, t) \pmod{p}\},$$

then the generalized Nagao's conjecture states the following: if the Jacobian variety of $\mathcal{X}$ has no subvariety over $\mathbb{Q}$, then

$$-\lim_{N\to\infty} \frac{1}{N} \sum_{p\leq N} -\frac{\log p}{p} \sum_{t=0}^{p-1} a_{\mathcal{X}_t}(p) \;=\; \operatorname{rank} J_{\mathcal{X}}(\mathbb{Q}(T)).$$

The conjecture is known for some settings, for example, Tate's conjecture implies Nagao's conjecture on elliptic curves. Assuming the generalized Nagao's conjecture, some work has been done to construct a hyperelliptic curve with a specific rank, but it is not generally known that for a given an arbitrary genus g and a positive integer r there is a hyperelliptic curve over $\mathbb{Q}(T)$ such that $\operatorname{rank} J_{\mathcal{X}}(\mathbb{Q}(T)) = r$.

Assuming the generalized Nagao's conjecture, we show the existence of hyperelliptic curves with a fixed genus g having certain ranks. Specifically, by constructing such a family of hyperelliptic curves, we prove that if $7 < r \leq 4g + 2$, there exists a hyperelliptic curve such that $\operatorname{rank} J_{\mathcal{X}}(\mathbb{Q}(T)) = r$. The proofs involve tools from algebraic geometry, Galois theory, and complex analysis.

2.23 Analytic Continuation of Plane-Wave Representations for Solutions to the Helmholtz equation on $\mathbb{R}^2$.

(ID = 36)

Justin E Shaw (UC Berkeley) justin.shaw@berkeley.edu
The Ohio State University [Mentor: Valentin Kunz]

Abstract

Mixed boundary value problems for the Helmholtz equation on the real 2-sphere are important for describing solutions to $3d$ scattering problems. We study plane-wave representations for solutions to these problems, and provide explicit formulas for their analytic continuation. In addition, we obtain a new proof of the analytic continuation formula for the Legendre function P_λ.

2.24 A Valsalva Maneuver Model to Phenotype Autonomic Dysfunction (ID = 37)

Parsa Khodabandehlou (North Carolina State University) pkhodab@ncsu.edu
North Carolina State University [Mentor: Mette Olufsen]

Abstract

Autonomic dysfunction (AD) involves failure of the sympathetic and parasympathetic nervous systems to control blood pressure (BP) and heart rate (HR) in response to an applied stress. AD is elusive due to vague symptoms, such as dizziness or fatigue, and comorbidities including Parkinson's and diabetes. Diagnosis of AD involves dynamic testing that measures BP and HR during head-up tilt, deep breathing, and the Valsalva maneuver (VM) (forced exhalation against a closed airway). The dynamic and varied outcomes of these tests make the associated HR and BP time-series data challenging to analyze. This study focuses on analyzing features from a computational VM model used to estimate parameters for more than 800 patients, many of whom exhibit missing or exaggerated dynamics. We incorporate shape statistics into the computational model and use sensitivity analysis to identify a set of influential parameters. This approach allows us to estimate model parameters effectively and with high certainty. Finally, using clustering on the estimated parameters, we expect to improve patient phenotyping, and therefore treatments.

2.25 The fourth and fifth coefficients of the $\mathfrak{sl}_3$ invariant on positive alternating links (ID = 39)

Joshua M. O'Connor (University of Kansas) j984o820@ku.edu
Angie Wu (Boston College) wuangi@bc.edu
Michigan State University [Mentor: Matthew Harper]

Abstract

The $\mathfrak{sl}_3$ link polynomial is a higher rank relative of the Jones polynomial which can be studied entirely diagrammatically using Kuperberg's web calculus. The invariant is known to detect whether a link is fibered and provides an obstruction to present positive links as positive braids. Building on the work of Harper and Kalfagianni, we construct formulae for the fourth and fifth coefficients of the quantum $\mathfrak{sl}_3$ link invariant on positive alternating links in terms of data read from the associated Seifert graph. Then, we give a characterization of the invariant on positive alternating fibered links in terms of elementary symmetric polynomials.

We also formulate the Garside element ($\Delta_3 \in B_3$) in terms of elementary webs, and give a closed formula for the polynomial on positive 3-braids. We outline additional results and work in progress towards a characterization of the $\mathfrak{sl}_3$ polynomial on all 3-braids.

2.26 Cardiovascular Modeling for Heart Failure Classification (ID = 41)

Logan Scott Lyon (North Carolina State University) lslyon@ncsu.edu
North Carolina State University [Mentor: Mette Olufsen]

Abstract

Despite considerable research and improvements in treatment, the heart failure (HF) epidemic remains a consistent public health challenge. HF with preserved ejection fraction (HFpEF), which accounts for over half of HF cases, remains difficult to diagnose due to the heterogeneity of the patient pool. Diagnostic guidelines define HFpEF patients as having a normal-appearing ejection fraction (EF) of over 50%, while still exhibiting the characteristic HF symptoms associated with HF with reduced EF (HFrEF). De-identified clinical data was collected from patients with right-heart catheterization (RHC), transthoracic echocardiogram (TTE), and MRI procedures. Using a mechanistic differential equation model of the cardiovascular system, we estimate unique parameter sets, which describe the cardiovascular

state of each patient, and can be used by clinicians to better phenotype HFpEF populations. We implement local and global sensitivity analysis and subset selection to determine which parameters are sensitive to the available data and can be uniquely identified. On these parameter sets, we conduct unsupervised machine learning to cluster HF patients in an effort to reveal underlying mechanistic phenotypes. These methods hold promise to enhance diagnostic accuracy of HF patients and uncover differences between HFpEF phenotypes.

2.27 How Lockdown Reshaped Twitter Posting Dynamics (ID = 42)

Sanay Salvi
Chester Wonchoba (Northwestern University) cwoncho@ncsu.edu
North Carolina State University [Mentor: Ana-Maria Staicu]

Abstract

The COVID-19 lockdowns had documented effects on mental health and social behavior. Twitter offers a precise, timestamped record of how individuals responded, but most studies reduce this to daily tweet counts and lose the finer rhythm of when and how people post. We develop a modeling framework to characterize individual posting dynamics at a finer temporal resolution than counts allow, identify common posting regimes and atypical users, assess whether lockdown altered posting structure beyond volume, and identify response phenotypes reflecting distinct behavioral shifts. The framework models posting as a point process that treats original tweets and retweets as two distinct, interacting behaviors. Its core is a bivariate Hawkes process whose fitted parameters provide the structural description we seek. Clustering baseline parameters identifies common posting regimes and atypical accounts; clustering pre-to-post parameter changes identifies response phenotypes. We extend the model to account for emotional content and user characteristics, and we assess its performance against simpler point processes that each drop one feature - either the distinction between tweet types or the bursty, self-reinforcing nature of posting itself. Fitting it separately before and after lockdown onset lets us measure how posting dynamics changed. We apply this methodology to roughly 400 Twitter accounts in Raleigh, NC, identified by their use of mental-health distress language during lockdown.

2.28 S^1-Fixed Loci of Hyperkähler ALE Spaces Corresponding to the Binary Dihedral Groups BD_n (ID = 43)

Haozhen Xu (Rice University) hx28@rice.edu
Rice University [Mentor: Jiajun Yan, Jo Nelson]

Abstract

Hyperkähler asymptotically locally Euclidean (ALE) spaces are a class of four-dimensional manifolds that admit three mutually compatible quaternionic complex structures and have been widely studied in symplectic geometry, algebraic geometry, and representation theory. Through the McKay correspondence, finite subgroups of $SU(2)$ determine hyperk"ahler ALE spaces. The computation of their S^1-fixed loci provides important input for geometric and topological computations such as Morse-theoretic and equivariant cohomological calculations. In our work, with Butera, Ruan, Ryang, and Yao under the mentorship of Nelson and Yan, we compute the S^1-fixed loci of the hyperk"ahler ALE spaces corresponding to the binary dihedral subgroups $BD_n \subset SU(2)$. We approach this problem by computing the level set of the corresponding hyperk"ahler moment map, which determines the manifold as a quotient space in terms of a representation-theoretic multigraph known as a quiver. We then analyze the quotient geometrically to identify the fixed-point set. Our method utilizes a computer program to compute and verify the fixed-point equations arising from the McKay quiver. We additionally prove an explicit geometric description of the fixed-point loci associated with the outgoing branches of the BD_n McKay quiver. This gives a concrete description of the S^1-fixed loci for the hyperk"ahler ALE spaces associated with the binary dihedral family.

2.29 Local topological order and boundary algebras for condensation in Levin-Wen systems (ID = 44)

Julia G. Circele (The Ohio State University) circele.1@buckeyemail.osu.edu
The Ohio State University
[Mentors: Jiaqi Lu, Melody Molander, David Penneys, and Greyson Wesley]

Abstract

Lattice models are discrete mathematical depictions of physical systems. In particular, the Levin-Wen system is a lattice model built from a unitary fusion category (UFC) such that on each vertex of the lattice there exists a collection of Hilbert spaces defined in terms of the UFC. There is a lattice model for anyon condensation in Levin-Wen systems given in [arXiv: 2303.04711]. In this talk, we will explain how this model satisfies the local topological order axioms from [arXiv: 2307.12552] and how the boundary algebra for this model is an "A-augmented" fusion spin chain, where A is a condensible algebra in the Drinfeld center of the UFC. This work is joint with Jiaqi Lu, Melody Molander, David Penneys, and Greyson Wesley.

2.30 Continuous spectra of a specific one-ended orbital Schreier graph of the Basilica group (ID = 46)

Luca Lamardo (Tufts University) luca.lomardo@tufts.edu
Isabel C.G. Walder (St. Mary's College of Maryland) iwalder@smcm.edu
University of Connecticut
[Mentor: Luke Rogers]

Abstract

We consider the Basilica group as described in [**?**] which is a foundational example in the theory of self-similar groups [**?**]. It is generated by a 3-state automaton acting on a binary tree. One way this is studied is through its Schreier graphs Γ_n, which are analogous to Cayley graphs restricted to the action on specific levels of the binary tree. More specifically, we can examine the Basilica group through the spectra of the Laplacian on these graphs.
Certain pointed limits of the Γ_n as $n \to \infty$, called orbital Schreier graphs, are classified by D'Angeli et al [**?**] and the Laplacian spectra of many of these were characterized in [2] by considering them to be blowup graphs in the sense of Strichartz [**?**]. However, there is a one-ended orbital Schreier graph to which past techniques are not applicable. This graph is of particular interest because it is similar to infinite graphs that have continuous Laplacian spectra [**?**] [**?**], which is significant in the physics of aperiodic media [3] [**?**] [1]. We will discuss our work aimed at establishing whether this specific orbital Schreier graph has continuous Laplacian spectra through an examination of the dynamics that would necessarily be satisfied by eigenfunctions and that arises from the self-similarity of the graphs.

References

[1] Michael Baake, David Damanik, and Uwe Grimm, *What is . . . aperiodic order?*, Notices Amer. Math. Soc. **63** (2016), no. 6, 647–650. MR 3495224

[2] Antoni Brzoska, Courtney George, Samantha Jarvis, Luke G. Rogers, and Alexander Teplyaev, *Spectral properties of graphs associated to the Basilica group*, Int. Math. Res. Not. IMRN (2026), no. 5, Paper No. rnag037, 34. MR 5041087

[3] David Damanik, Mark Embree, and Anton Gorodetski, *Spectral properties of Schrödinger operators arising in the study of quasicrystals*, Mathematics of aperiodic order, Progr. Math., vol. 309, Birkhäuser/Springer, Basel, 2015, pp. 307–370.

2.1 Flat conditional exponential directed last passage percolation under a one-point upper large deviation event
(ID = 47)

Rishi Dadlani (University of Pennsylvania) rdadlani@sas.upenn.edu
Shaotian Sun (University of Michigan, Ann Arbor) shaotian@umich.edu
University of Michigan [Mentor: Jinho Baik]

Abstract

Exponential directed last passage percolation (Exponential DLPP) is a random maximization problem. Each element of a two-dimensional integer lattice carries an exponential random variable, and among all directed up-right lattice paths, one asks which collects the largest total weight. The model has interpretations ranging from random growth to tandem queues to interacting particle systems. We study it under a flat initial condition, where the starting point is allowed to vary, and ask how the model behaves conditioned under an atypically large last passage time to a given point. Closely related questions were recently answered for a different geometry, the step initial condition, where the initial point was fixed. Guided by these results, we develop heuristic arguments to predict how the flat model should behave under the same conditioning, and prove corresponding fluctuation results in certain regimes. Our method is to analyze exact multi-point formulas using steepest descent. The flat setting introduces a new coupling not present in the step case, and our calculations suggest that its conditional fluctuations should again be built from Brownian bridges coupled through an additional shared Gaussian component.

2.2 Variants on the Zeckendorf Game
(ID = 51)

Lillian Cao (University of Cambridge) lc2007@cam.ac.uk
Naida Gavranović (University of Cambridge) ng558@cam.ac.uk
Emily Upton emilyupton220@gmail.com
Williams College [Mentor: Steven Miller]

Abstract

Zeckendorf's Theorem states that every integer can be uniquely expressed as the sum of non-consecutive Fibonacci numbers, a concept that forms the basis of the Zeckendorf Game. In this two-player game, participants take turns applying two types of moves to an initial integer n until it reaches its Zeckendorf decomposition, and the first player to complete this decomposition wins. Previous work showed the game always terminates, and if $n > 2$ that Player 2 has a winning strategy (although this proof is non-constructive proof and involves a parity steal).

We build upon this framework through two main avenues of analysis. First, we study a simpler base-2 decomposition game where players combine two 2^k values into a single 2^{k+1} value. For this system, we introduce a highest summand variant: the first player to generate the largest possible summand wins. We derive a recurrence relation that strictly determines the winning player, and discuss the consequences. Second, returning to the Fibonacci framework, we give a non-constructive proof that the second player also has a winning strategy in the highest summand variant of the Zeckendorf Game, alongside other game modifications where winning is conditional only on certain game states. In ongoing work we are trying to make the proof constructive.

2.3 Enumerating Weak Order Unit Fubini Rankings
(ID = 52)

Eban M. Barrett (The University of Alabama in Huntsville) emxb2006@gmail.com
Maritza A. Sanchez (University of Puerto Rico in Ponce) marisanchez92204@gmail.com
Tung Vo (University of St. Thomas) vo.tung@stthom.edu
University of Wisconsin-Milwaukee [Mentor: Pamela Harris]

Abstract

We study the intersections of two subsets of parking functions: Fubini rankings and unit interval parking functions. We show that they can be characterized by the behavior of their weakly increasing elements. We use our characterizations to enumerate their intersections and weakly ordered counterparts, yielding known sequences such as the Fibonacci numbers. In particular, we show that the weakly increasing and weakly decreasing unit Fubini Rankings and weakly decreasing unit interval parking functions are enumerated by the Fibonacci numbers.

2.4 Random Fibonacci Tilings (ID = 54)

Emily Upton emilyupton220@gmail.com
Connor Yau (Williams College) cy10@williams.edu
Williams College [Mentor: Steven Miller]

Abstract

The Fibonacci spiral is typically created from a simple square joining rule; that is, begin with a unit square and continuously attach squares whose side lengths follow the Fibonacci sequence, and in the standard construction, the choices of direction produce the familiar spiral and a tiling of the plane. This project studies what happens when the construction is randomized.

At each stage, the next square is attached along the predetermined horizontal or vertical axis, but at each stage a fair coin determines which of the two possible directions is chosen; left or right in the horizontal case and up or down in the vertical case. Then for a fixed lattice point (i, j) we ask when this random Fibonacci tiling first covers that point and how the expectation time depends on its location. To do so, we consider tracking the distances from the origin to the four edges of the growing rectangle and determine that these distances are governed by random subset sums of even and odd indexed Fibonacci numbers. This provides us with recursive formulas for the relevant hitting probabilities and leads to an expression for the expected number of steps before the first time covering any particular lattice point.

2.5 Approximation of the Strong Monotonicity Constant of a Function (ID = 57)

Gwendolyn M. Burlingame (Temple University) benburlingame56@gmail.com
The Ohio State University [Mentor: Marjorie Drake]

Abstract

For a finite set $E \subset \mathbb{R}^n$, where $\#E = N$, directly calculating the strong monotonicity constant of a function $f : E \to \mathbb{R}^m$ requires $O(N^2)$ computations. Fefferman and Israel use the Well-Separated Pairs Decomposition (WSPD) to approximate the Lipschitz constant of f with $O(N)$ computations. Utilizing the WSPD, we approximate the strong monotonicity constant of a function with $O(N)$ computations. Based on this computation, we explore almost optimal monotone, Lipschitz extensions of the function f.

2.6 Concavity structure of fractional differential equations and zeros of Mittag–Leffler functions (ID = 62)

Jamie L. Zhang (University of Dayton) lzhang4@udayton.edu
(University of Dayton [Mentor: Yulong Li]

Abstract

In classical calculus, the concavity of a function is determined by the sign of its second derivative. In this paper, we develop a real-variable framework for studying concavity in the nonlocal setting of Riemann–Liouville fractional derivatives of order $1 < \alpha < 2$. Under homogeneous boundary conditions, we show that the sign of the fractional derivative alone does not determine global concavity. Instead, the concavity exhibits an intrinsically nonlocal behavior: if the fractional derivative is non-decreasing, then the solution is strictly concave up; whereas if it is nonnegative and sufficiently decreasing near the right endpoint, then the solution is strictly concave up near the left endpoint and strictly concave down near the right endpoint. Consequently, every such solution possesses at least one inflection point. As an application, we prove the existence of a positive real zero of $E_{\alpha,\beta-2}(-z)$ for $1 < \alpha < 2$ and an admissible range of β, establish its comparison with the first positive real zero of $E_{\alpha,\alpha}(-z)$, and identify a new ordering among the first positive real zeros of $E_{\alpha,\beta}$ as β varies. More broadly, our approach provides a direct real-variable method for locating real zeros of special functions through the geometric properties of fractional differential equations.

2.7 Sensor sensitivity in several settings (ID = 80)

Christopher Householder christopherlancehousholder@gmail.com
Williams College [Mentor: Steven Miller]

Abstract

Many problems in signal recovery, sampling, and learning depend on understanding when a function is concentrated in frequency space. Classical sparsity assumptions are often too rigid for smooth continuous data and especially so after the data is discretized onto a finite grid. Recent work develops the notion of the Fourier Ratio which uses the ℓ^1 and ℓ^2 norms of the Fourier transform as a measure of spectral compressibility to give a sparsity-free way to measure the effective size of Fourier support. We further study these discretization estimates in several directions. First, we prove Fourier Ratio bounds for periodic functions on $([0, 1]^2)$ under new weaker C^1 regularity assumptions. Then we introduce a small pre-discretization perturbation in which the continuous function is shifted before being sampled on the grid. We use this shift to choose a discretization with better L^2-mass control which leads to a sharper Fourier Ratio bound. We then go further by proving an analogous estimate on the sphere (S^2) using spherical harmonic decomposition. Finally, we extend the Fourier Ratio idea to compact Riemannian manifolds using Laplace-Beltrami spectral decompositions and prove corresponding signal recovery results for bandlimited functions from random samples. These estimates give a more flexible version of the original discretization method and suggest further direction for smooth data sampling under less rigid geometric assumptions.

2.8 Expected Moments of the Spectral Measure for 2-Level Bipartite Graph Adjacency Matrices (ID = 90)

Naida Gavranović (University of Cambridge) ng558@cam.ac.uk
Connor Yau (Williams College) cy10@williams.edu)
Williams College [Mentor: Steven Miller]

Abstract

Random matrix theory provides a way to understand the behaviors of various systems in mathematics. In particular, eigenvalue distributions from random matrix ensembles have been connected to the distribution of energy levels in heavy nuclei and to the spacings between zeros of the Riemann zeta function. These recurring properties suggest a form of universality and make random matrix theory a useful framework for studying spectral behavior in a wide range of settings.

Graphs provide an important setting for applying these ideas because their adjacency matrices encode important structural information about a graph, like graph connectivity or counting walks. McKay proved that the limiting spectral distribution of random d-regular graphs converges to the Kesten–McKay law, while Leo Goldmakher, Cap Khoury, Steven Miller, and Kesinee Ninsuwan extended this setting by introducing random edge weights and studying the resulting eigenvalue distributions.

We study the moments of composite graph ensembles constructed by starting with a b-regular bipartite graph between two vertex sets, where each part has individually a-regular graph structure, with $a + b = d$.

Using the combinatorial trace method, we enumerate closed walks of length k to derive explicit formulas for the expected spectral moments $\mathbb{E}[\mu_k]$. Specifically, we analyze how the parameters a and b determine how the spectral measure of the graph ensemble converges to Kesten–McKay.

2.9 Decoding Polar Affine Grassmann Codes (ID = 100)

Jaime Jimenez Arevalo arevalojjaime@gmail.com
University of Wisconsin-Milwaukee [Mentor: Pamela Harris]

Abstract

An Affine Grassmann code is a linear code obtained by evaluating the minors of a generic $l \times m$ matrix. By imposing an orthogonality condition on elements of the grassmannian, we obtain the Affine Polar Grassmann code as a subspace. In this talk, we will investigate possible decoding algorithms and methods for determining parity check equations for the Affine Polar Grassmann code. We will look at the special case of invertible symmetric 2×2 matrices, and their behaviors under a specific action of the general linear group.