Young Mathematicians Conference (YMC) 2025 Abstract Book

July 29, 2025

Contents

1 Reasoning Through Contradiction: A Paraconsistent Categorical Structure (ID = RR26231136)

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Independent Researcher [Mentor: n/a n/a]

Abstract

This project presents a fibered, paraconsistent categorical structure to allow for reasoning in inconsistent settings where contradictory beliefs may arise. Each morphism (corresponding to a deductive step) carries *descent data* that tracks how a result was derived, enabling safe recovery of classical truth when possible.

While creating this system, I discovered situations where naive rules for combining results, like assuming disjoint descent data means independence, fail. To address this, I have introduced structured descent tracking using directed graphs, along with formal conditions that control when results from different epistemic contexts can safely be glued together.

The system pulls ideas from category theory, type theory, and paraconsistent logic. Parts are implemented in the proof assistant Agda, and I am working toward eventually organizing it with higher-categorical structures like ∞-groupoids.

2 Towards Thermodynamic Formalism for Countable-State Shift Spaces With Specification

(ID = PA28113129)

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Abstract

We define and discuss specification properties for countable-state shift spaces. This is based on a definition for flows on locally compact complete separable metric spaces by Climenhaga, Thompson, and Wang. We prove the variational principle for pressure and discuss the ongoing development of thermodynamic formalism in this setting. We remove a local compactness assumption that is present in the work of Climenhaga, Thompson and Wang.

3 Magic Squares of Squares

(ID = SB29214210)

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Abstract

Magic squares – $n \times n$ grids of integers whose rows, columns, and diagonals all add up to the same integer – have been of mathematical interest for millennia. Magic squares are easy to parametrize with linear algebra, but what about magic squares whose entries are a specific power? The first instance of a magic square of powers was the 4×4 square of squares

68^2	29^2	41^2	37^2
17^2	31^2	79^2	32^2
59^2	28^2	23^2	61^2
11^2	77^2	8^2	49^2

with row sum 8515 due to Euler (1770). Recently, Rome and Yamagishi (2024) showed that an $n \times n$ magic square of kth powers exists for all $n \geq 4$ and all $k > 0$. However, this leaves the 3×3 square of squares case open. Notably, Lucas (1891) studied 3×3 squares of squares, but could not solve the problem before he passed.

When viewed geometrically, the algebraic variety that the equations for a 3×3 square of squares determine is a singular surface of general type V_{ms}. From this point of view, the problem is much stronger than the Bombieri-Lang conjecture for V_{ms}, explaining its difficulty. Our work produces major steps in unraveling V_{ms}'s geometry. We calculate the Hodge numbers of its resolution $\widetilde{V_{ms}}$ by studying its differential properties in a family of complete intersections. We prove that it has torsion-free and maximal rank Neron-Severi group by studying its singular cohomology and finding an explicit basis for its Picard group. This culminates in computing V_{ms}'s algebraic Brauer group. Finally, we outline a potential approach to solving the problem through methods for studying the Brauer-Manin obstruction on quasiprojective subvarieties that we will explore in future work.

4 On the uniqueness of positive solutions for classes of two-point boundary value problems when a para

(ID = WC30104855)

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Abstract

We consider positive solutions to boundary value problems of the forms:

$$\begin{cases} -u'' = \lambda f(u); \ (0,1) \\ -u'(0) + bu(0) = 0 \\ u'(1) + bu(1) = 0 \end{cases} \tag{1}$$

$$\begin{cases} -u'' = \lambda g(u); \ (0,1) \\ u(0) = 0 \\ u'(1) + bu(1) = 0 \end{cases} \tag{2}$$

where λ and b are positive parameters. We will discuss four examples of f, and an example of g for which the boundary value problems (1) and (2) respectively have multiple positive solutions for a finite range of λ, while there exists a $\lambda_b > 0$ such that they have a unique positive solution for $\lambda > \lambda_b$. We discuss the evolution of these $\lambda_b's$ when b varies via a quadrature method, which traces its origin to the work by T-Laetsch, and mathematica computations. We demonstrate that λ_b is an increasing function of b and converges to a finite λ_D as $b \to \infty$. Note that this implies that both (1) and (2) have a unique positive solution for $\lambda > \lambda_D$, which is independent of b. Here, λ_D is such that (1) and (2) with Dirichlet boundary conditions at both ends have a unique positive solution for $\lambda > \lambda_D$. (Joint work with N. Fonseka and R. Shivaji).

5 Choice of Strategy in Deterministic Zeckendorf Games

(ID = WR30134514)

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Abstract

Zeckendorf proved that every positive integer m can be uniquely written as a sum of non-consecutive Fibonacci numbers F_n, where $F_1 = 1, F_2 = 2$, and $F_{n+1} = F_n + F_{n-1}$. We refer to this representation as the Zeckendorf decomposition of m. Motivated by this result, Baird-Smith et al. created a two-person Zeckendorf Game, played on a series of columns labelled in order with the Fibonacci numbers. Beginning with n tokens in the F_1 column, players alternate moves by combining ($F_k \wedge F_{k+1} \mapsto F_{k+2}$) or splitting ($F_1 \wedge F_1 \mapsto F_2,\ F_2 \wedge F_2 \mapsto F_3 \wedge F_1$, or $2F_k \mapsto F_{k-2} \wedge F_{k+1}$ for $k \geq 3$) tokens. After any legal move, the sum of the tokens weighted by their columns is always m. The last player to move wins, and they proved the game always terminates in m's Zeckendorf decomposition.

Baird-Smith et al. provide a non-constructive proof that Player 2 has a winning strategy whenever $m > 2$; constructive winning strategy is unknown. In order to simplify analysis of gameplay, Li et al. considered four deterministic variants of the game, in which there is a prescribed order on which available move a player must take: combine largest, combine smallest, split largest, or split smallest. We consider a game in which m is fixed and Players 1 and 2 are each offered a choice amongst the four deterministic variants, without knowledge of the variant their opponent will select. This offers 16 pairs of strategies. We prove results on who wins based on m and which pair is chosen and compute the expected payoff for each player of selecting a particular variant.

6 Centered Moments of the Weighted One-Level Density of $GL(2)$ L-Functions

(ID = MV30153903)

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Abstract

In 1972, Hugh Montgomery observed a groundbreaking connection between the zeros of the Riemann zeta function and the eigenvalues of large, random Hermitian matrices (which had already successfully modeled the energy levels of heavy nuclei). He proved that the pair correlation of the scaled zeros matches the pair correlation of the scaled eigenvalues in their respective limits. Many results followed from this fascinating correspondence by generalizing from the zeta function to L-functions (which encode information about various important arithmetic problems). The initial agreements were for zeros far from the central point $s = 1/2$, where the spacing between adjacent zeros tends to zero, so only one modular form provides sufficient quantities to average. For many problems (such as the Birch and Swinnerton-Dyer conjecture on the ranks of elliptic curves), it is important to understand the behavior of the low-lying zeros, i.e. those near $1/2$. This led Katz and Sarnak in 1998 to create a new statistic, the n-level density (or equivalently the n^{th} centered moment), where now we average over a family of similar L-functions. They conjectured agreement between these statistics and the eigenvalues of certain ensembles of random matrices. This conjecture is known for several families in restricted ranges; sometimes the forms are not weighted equally to facilitate the application of averaging formulas.

Initially, these weights were for technical convenience, and papers showed how to remove them through involved sieving. In 2018, Knightly and Reno proved that the exact nature of these symmetries depends on how these low-lying zeros are weighted. They observed both orthogonal and symplectic symmetry in the 1-level density of families for different choices of weights. We prove the same dependence of symmetry on weights in the n^{th} centered moments of the 1-level densities for two different families of holomorphic cusp newforms, given suitably restricted test functions. We calculate these moments using vertical equidistribution results for weighted Hecke eigenvalues to get a multi-dimensional integral. We then convert the integral to the one-dimensional integral appearing in Knightly and Reno using a change-of-variables technique of Hughes and Miller for moment calculations and then show agreement through involved combinatorics.

7 Using Gowers Norms in Improving Signal Recovery Conditions

(ID = SP30160408)

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Abstract

When transmitting signals, we often send the Fourier transform instead of the raw signal, as it's usually more convenient and efficient. However, often the channel is noisy, causing some of the frequencies to be unobservable and preventing accurate reconstruction. Recently Iosevich and Mayeli analyzed the sparse, finite, binary case with signal $f : \mathbb{Z}_N^d \to \{0, 1\}$, $\hat{f}$ suitably normalized, and $S \subset \mathbb{Z}_N^d$ the set of unobserved frequencies. They prove many recovery estimates that ensure exact recovery after applying simple rounding algorithms. Specifically, they use restriction theory to show that if the additive energy

$$|\{(x, y, x', y') \in U^4 : x + y = x' + y'\}|$$

is sufficiently small for every $U \subset S$, then f can be recovered exactly.

We extend the use of additive energy to so-called higher Gowers norms, a recent breakthrough from additive combinatorics. The second Gowers norm coincides with additive energy, and higher norms measure the arithmetic structure within the set. We prove, in analogy with the above result, that if the third Gowers norms of all $U \subset S$ are suitably small, we get a better recovery estimate on f.

8 Applications of Gabor frames in signal recovery

(ID = KS01093327)

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Abstract

Central to signal processing is the problem of exact signal recovery - the reconstruction of a signal from incomplete data. Donoho and Stark used Fourier uncertainty principles to establish conditions for the exact recovery of a signal from its discrete Fourier transform. Using restriction theory and the Salem set mechanism from harmonic analysis, Iosevish and Mayeli established sharper uncertainty principles and provided a simple recovery mechanism in the case of binary signals, via the DRA algorithm.

A closely related problem is the imputation of missing data in time series, which is central to signal processing, statistics, and data science. In recent work by Burstein, Iosevich, Mayeli, and Nathan, the classical L^1 minimisation method for signal recovery was adapted to tackle this problem, leveraging results of Bourgain and Talagrand.

We extend these results using the Gabor transform, a powerful tool in time-frequency analysis, as a substitute for the usual Fourier transform. An orthogonal Gabor basis for $L^2(\mathbb{Z}_N^d)$ takes the form

$$\{g(x-a)\chi(x\cdot b)\}_{(a,b)\in S},$$

where g is a window function giving higher weight to the signal near the time being analyzed, and $\chi(t) = e^{2\pi it/N}$. New features emerge when applying the Gabor transform to exact recovery and data imputation, allowing us to improve upon previous results.

9 More Sums than Differences in F_2

(ID = KA01101131)

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Abstract

For a group G, we define the right quotient set and the left quotient set as follows

$$AA^{-1} := \{a_1 a_2^{-1} : a_1, a_2 \in A\},$$
$$A^{-1}A := \{a_1^{-1} a_2 : a_1, a_2 \in A\}.$$

We examine the relationships between the left and right quotient sets. If G is an abelian group, then these sets are equal, but subtleties arise in non-abelian settings, as these sets may not have the same cardinality. Tao remarked that the cardinality difference $|AA^{-1}| - |A^{-1}A|$ may be arbitrarily large for certain groups.

We first give explicit constructions of sets A where this difference attains every possible integer, proving that the difference can be any possible value if G has elements of order 2.

We also find the minimum cardinality of A so that the difference between the cardinalities of the left and right quotient sets is nonzero, depending on the existence of order 2 elements in G.

To prove these results, we construct a graph called the difference graph D_A that encodes equality in the right quotient set. Similarly, $D_{A^{-1}}$ encodes equality in the left quotient set. By observing an isomorphism of edges in D_A and $D_{A^{-1}}$ and counting connected components, we are able to prove the results above. In the free group on two generators, we can prove that the difference $|AA^{-1}| - |A^{-1}A|$ is always even. We explicitly construct subsets of F_2 that achieve every even integer. In the infinite dihedral group $D_\infty \cong \mathbb{Z} \rtimes \mathbb{Z}/2$, we prove that every integer difference is achievable, using the results of Martin and O'Bryant on the cardinality differences of sum sets and difference sets in $\mathbb{Z}$.

10 Explicit generators of the space of modular forms

(ID = SA01111924)

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Abstract

One of the main reasons for the importance of modular forms in number theory is the fact that spaces of modular forms are spanned by forms with rational Fourier coefficients. These Fourier coefficients are often arithmetically interesting. For example, divisor functions appear in the Fourier coefficients of Eisenstein series. On the other hand, spaces of modular forms have natural rational structures defined by the rationality of *periods*. These are defined as follows: let S_κ denote the space of cusp forms of weight κ and level one. For each $0 \leq t \leq \kappa - 2$, the t-th period of $f \in S_\kappa$ is given by

$$r_t(f) := \int_0^{i\infty} f(z) z^t dz = \frac{t!}{(-2\pi i)^{t+1}} L(f, t+1).$$

The theory of periods of modular forms was developed by Eichler, Shimura, and Manin, and it has been extensively studied by various authors such as Kohnen and Zagier. One of the fundamental results is the Eichler-Shimura isomorphism, which states that odd periods $r_1, r_3, \ldots, r_{\kappa-1}$ span the vector space S_κ^*, and so do even periods $r_0, r_2, \ldots, r_{\kappa-2}$. In this study, we find explicit subsets of periods that span S_κ^*. This result is achieved by showing the non-singularity of a matrix that arises from the Eichler-Shimura relations. As an application, we find a set of generators of S_κ consisting of Rankin-Cohen brackets of Eisenstein series. This is joint work with Yanhui Su and Hui Xue.

11 Turning Tables: How Degeneracy Tables Encode Geometric Information

(ID = CA01161203)

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Abstract

In 1966 Mark Kac made famous the question: can one hear the shape of a drum? In other words, how much geometric information can one discern about a manifold based on the spectrum of the Laplace operator on that manifold? Over CR manifolds the complex analogue of the Laplace operator is the Kohn-Laplacian, $\Box_b$. We study the spectrum of $\Box_b$ for quotient spaces of unit spheres in $\mathbb{C}^n$ under the action of a finite group $G < U(n)$. In these G-invariant spaces, the spectrum of $\Box_b$ corresponds exactly with the vector spaces $\mathcal{H}_{p,q}^G$ of harmonic homogeneous complex polynomials in z and $\overline{z}$ of bidegree p, q. These in turn are characterized by spherical harmonics that are invariant under the action of G. We prove that, in $\mathbb{C}^2$, as long as G is a subgroup of $SU(2)$, the dimension of $\mathcal{H}_{p,q}^G$ will be a function of only $p + q$. We prove the converse for lens spaces, a specific type of sphere quotient defined using a cyclic group. That is, if $G < U(2)$ is not a subgroup of $SU(2)$, then the dimension of $\mathcal{H}_{p,q}^G$ is not a function of $p + q$. We are also working to generalize these results to higher dimensions.

12 Matrix Decomposition for Complex Vortex Structure Characterization (ID = TJ01231019)

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Yale University [Mentor: Samuel Panitch]

Abstract

The accurate characterization of complex spatiotemporal structures, such as coherent vortices in high-speed turbulent boundary layers, poses a significant challenge due to the high dimensionality and nonlinear nature of the governing Navier-Stokes equations. To circumvent these issues, matrix decomposition related techniques, e.g., Principal Component Analysis (PCA), were developed. Through PCA, a matrix can be decomposed into its temporal coefficients and spatial modes. The main issue encountered when using a dimensionality reduction technique, such as PCA, is the appearance of pseudo-vortices — a truncation error from the unresolved high frequency spatial modes. In this study, we explore various matrix decomposition techniques to reconstruct complex vortex structures while minimizing pseudo-vortices, coupling modal decomposition techniques with a novel vortex identification strategy.

We extend the use of Nonnegative Matrix Factorization (NMF) into the domain of Fluid Dynamics and compare NMF to PCA while utilizing normalization techniques to yield the most optimal finite-dimensional approximation of spatial modes and their corresponding temporal coefficients. **Our main conclusion is that NMF minimizes pseudo-vortices better than conventional PCA methods employed in the past.** Our methodology establishes a computationally efficient platform for investigating vortex dynamics, offering substantial reductions in data dimensionality while preserving essential features of turbulent transport.

13 Matrix Product Operators for Honeycomb Matchings

(ID = LA02025009)

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Abstract

The dimer model, which studies perfect matchings on graphs, is a central object in rigorous statistical mechanics, with deep connections to combinatorics and probability. Of particular interest are dimers on the honeycomb lattice, where the enumeration of dimer coverings may be "exactly solved" using Kasteleyn's method or the coordinate Bethe ansatz. Quite surprisingly, to our knowledge, there is no honeycomb exact solution using the matrix product operator (MPO) formalism (via the Jordan-Wigner transformation), which was utilized by Lieb (1967) to exactly solve the dimer model on the square lattice.

We address this gap by exactly solving the dimer model on the honeycomb lattice (on a bi-infinite cylinder) via MPOs. Our transfer matrix construction differs slightly from that of the coordinate Bethe ansatz approach, which leads to important implications regarding boundary conditions. In particular, while "slight" changes to boundary conditions on the square lattice do not lead to differences in the thermodynamic limit, we explicitly demonstrate that the same cannot be said for honeycomb dimers. Our solution may be viewed as a natural extension of Lieb's method on the square lattice, allowing us to conveniently construct an MPO for the unsolved monomer-dimer model on the honeycomb lattice. The leading eigenvalues of our monomer-dimer MPO converge remarkably quickly compared to the square lattice case. This rapid convergence allows us to numerically compute asymptotics (e.g. the entropy per site) to a relatively high precision without having to resort to fancier constructions, such as Baxter's corner transfer matrices.

14 A simplified nonlinear mathematical model for blood flow in an aneurysm (ID = SP02090042)

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[Mentor: Vani Cheruvu]

Abstract

Rupture of a brain aneurysm (subarachnoid hemorrhage) can be catastrophic. In the literature, there are few studies based on mathematical models of blood flow in and around an aneurysm. G. Austin, Nikolov et al., J.J. Nieto, and A. Torres modeled this using variants of the Duffing equation. Our previous attempts to find semi-analytical solutions to these equations using Adomian and Laplace-based Adomian Decomposition Methods have some limitations, including a smaller radius of convergence for the series solutions. This research investigates the numerical solution and qualitative analysis of the variants of the Duffing equation.

15 Importance Sampling Methods for Tail Risk Estimation in Gaussian Copula Models (ID = WL02105118)

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Abstract

Estimating risk measures like Value-at-Risk and Expected Shortfall at high confidence levels poses challenges due to the rarity of large losses. Naive Monte Carlo simulation suffers from high variance in these cases. In this project, we study importance sampling methods to improve efficiency in estimating these tail probabilities under a conditional Gaussian copula model for correlated defaults. Specifically, we implement and analyze a two-step Exponential Twisting (ET) algorithm and compare it to baseline Monte Carlo across varying portfolio structures. We assess performance using numerical simulations, focusing on variance reduction and stability of likelihood ratios. Preliminary findings show that, when tuned appropriately, the ET method achieves significant variance reduction (often by an order of magnitude), making it a viable approach for rare-event estimation.

16 Structure and Dynamics of Laplacian Eigenfunctions on the Basilica Fractal (ID = RS02105402)

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University of Connecticut [Mentor: Luke Rogers]

Abstract

We study the Basilica group, a self-similar group generated by a 3-state automaton. Grigorchuk and Zuk [1], and later Bartholdi and Virag [2], established the group as the first example of an amenable but not subexponentially amenable group, and in the work of Nekrashevych [3], it occurs as the simplest nontrivial example in his theory of iterated monodromy groups.

In this talk, we investigate the spectral properties of the Schreier graphs of the Basilica group. These were first considered in [4], which revealed a family of weighted Laplacians on these graphs whose spectra are invariant under a two-dimensional dynamical system. A recent analysis of this dynamical system by Dang, Grigorchuk, and Lyubich [5] yields results on limiting properties of these spectra, obtained with respect to the conformally-invariant Laplacian in [6]. Our interest is in describing the structure of the eigenfunctions for this sequence of graphs, for which purpose we note that the dynamics from [4] correspond to a version of the notion of spectral self-similarity used by [7]. We develop an alternative description of the dynamical system in [4] and study the corresponding dynamics for the eigenfunctions. Moving forward, we aim to determine whether certain orbital Schreier graphs considered in [8] have singular or absolutely continuous spectra.

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REU Team Members

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17 The Ordered Zeckendorf Game

(ID = WC02111517)

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Abstract

The Fibonacci numbers $\{F_n\}$, where each number is the sum of the two preceding ones, is one of the most well-known sequences in mathematics. Zeckendorf proved that every positive integer can be written uniquely as a sum of non-consecutive Fibonacci numbers. This result has inspired a two-player game: starting with a multiset of n copies of $F_1 = 1$, players take turns applying the rule $F_i + F_{i+1} = F_{i+2}$ to combine terms and $2F_n = F_{n+1} + F_{n-2}$ to split, with the last to move winning. Every game ends in the Zeckendorf decomposition of n, and a non-constructive argument shows Player 2 has a winning strategy for $n > 2$.

We introduce a new variant where the order of Fibonacci numbers is preserved and operations are restricted accordingly. Players now work with an ordered list of n ones and can perform three types of moves: merging adjacent Fibonacci numbers using the recurrence relation, splitting certain pairs into smaller terms, and swapping adjacent terms to rearrange the list. This ordered version is more natural for algorithmic implementation and introduces richer structural complexity, as the state space becomes more nuanced and sensitive to sequence configuration. It also models situations where operations are inherently sequential or locally constrained—features common in computer science and game theory.

We prove that the game always terminates by defining and analyzing an appropriate monovariant (a quantity whose magnitude moves in only one direction as the game evolves). We also determine how long the game can last: the number of moves ranges from a linear lower bound of $n - Z(n)$ to a quadratic upper bound of $\frac{n(n-1)}{2}$, where $Z(n)$ is the number of terms in n's Zeckendorf decomposition. This ordered version presents new strategic and combinatorial challenges and opens up further directions in the study of number representations, discrete dynamics, and algorithmic gameplay.

18 Proper Holomorphic Maps Invariant under Finite Complex Reflection Groups (ID = PM02112915)

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Abstract

A classical problem in complex analysis is to study proper holomorphic maps between domains. In the case of maps between balls of different dimensions, there are often many inequivalent maps, and hence, we are interested in studying the subclass of rational maps that have symmetries (i.e. maps that are invariant under a finite subgroup of $U(n)$). D'Angelo and Lichtblau completely characterized which fixed-point-free groups arise in this context. Moreover, given any of these finite groups $\Gamma < U(n)$, they construct a Γ-invariant, proper polynomial map between balls. We are interested in the properties of such maps for groups that do have fixed points. Using the Shephard-Todd classification of finite unitary complex reflection groups, we study the target dimension of the D'Angelo-Lichtblau construction.

19 Spirals and the Dynamics of Deep Diagonal Maps

(ID = ZZ02113339)

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Brown University [Mentor: Richard Schwartz]

Abstract

Given a polygon, one can define the *deep diagonal map* T_k, which connects its k-th diagonals and intersects them successively to produce a new polygon. The case T_2, known as the *pentagram map*, is a well-studied discrete dynamical system with connections to symplectic geometry and cluster algebra. The map T_2 preserves convexity, and its orbits in the moduli space of projectively equivalent n-gons have compact closure. For $k \geq 3$, however, the maps T_k generally do not preserve convexity, and their dynamics remain less understood. We introduce new geometric structures, called *type-*α and *type-*β k*-spirals*, which are preserved under the action of T_k. Motivated by computer experiments, we derive a rational formula for T_3, which generalizes previous formulas obtained from cluster algebra. We then use this formula to prove that the orbits of type-α and type-β 3-spirals under T_3 have compact closure in the moduli space of k-spirals of the corresponding types. The talk will include live demonstrations using a web-based visualization program developed by the presenter that animates polygon orbits under T_k.

20 Gamma Factors for Triple Tensor Product Representations over Finite Fields (ID = BT02130151)

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[Mentor: Elad Zelingher]

Abstract

The representation theory of $\mathrm{GL}_n(k)$, where k is a local field, is crucial to number-theoretic questions, particularly concerning the local Langlands conjectures. In this work, we focus on finite-field analogs of the local Langlands theory for $n = 2$. While the classification of irreducible representations of $\mathrm{GL}_2(\mathbb{F})$, where $\mathbb{F}$ is a finite field, is known, questions remain about how certain operations like the tensor product behave under the local Langlands correspondence. To explore this, we attach local factors encoding various properties of the representation. Specifically, we investigate a construction of the gamma factor corresponding to the triple tensor product of three irreducible representations of $\mathrm{GL}_2(\mathbb{F})$. Our work continues Elber and Lheem's research on gamma factors for $\mathrm{GL}_2(\mathbb{F}) \times \mathrm{GL}_2(\mathbb{F}) \times \mathrm{GL}_2(\mathbb{F})$ by developing explicit expressions for the gamma factor in terms of Bessel functions and using this to prove a multiplicative property.

21 On the Identifiability of Leak Parameters in Linear Compartmental Models (ID = KT02133048)

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Abstract

In many applications, including in ecology and pharmacokinetics, linear compartmental models are used to model transfer between "compartments" which may represent populations, drug concentration, etc. Such models are represented by directed graphs in which the edges represent the transfers between compartments. An important feature of such models is the identifiability degree, which summarizes the extent to which it is possible to recover the transfer rates from noiseless experimental data. More precisely, the identifiability degree of a parameter is 1 if the transfer rate can be recovered uniquely, and is greater than 1 if the transfer rate can be recovered only up to a finite set (this size is equal to the degree).

In this presentation, we investigate the effects of adding leaks (edges directed out of the model) on the identifiability degree. We show that in a model represented by a strongly connected graph, if exactly one leak is in the same compartment as an output, then that leak parameter is uniquely identifiable. We investigate improvements to this result, looking at the preservation of the identifiability degree of the non-leak parameters and the applicability to non-strongly connected graphs, like in the case of directed path models.

22 Numerical Investigation of Partial Sums of Random Multiplicative Functions (ID = QK02145810)

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Ohio Wesleyan University [Mentor: Nick Geis]

Abstract

The Möbius function, $\mu(n)$, and other multiplicative functions are central objects in number theory. Recently, there have been several breakthroughs in number theory where proof techniques were motivated by those for probabilistic models for multiplicative functions. One such model is called *random multiplicative functions* (RMFs). An RMF, $f(n)$, is defined as a sequence of random variables where $f(p) = \pm 1$ with equal probability for each prime p; $f(n) = \prod_{p|n} f(p)$ for each square-free n; and $f(n) = 0$ otherwise. This mimics the definition for $\mu(n)$. Although RMFs have been investigated thoroughly using analytic methods within the last decade, there is significantly less work based on numerical methods.

We use `C++` to extensively perform Monte Carlo simulations of partial sums of RMFs for $n \leq 10^9$. Rather than studying the sizes of these sums, we instead analyze other properties like the growth rates of the number of sign changes and zeroes, as well as the proportion of sign changes to zeroes. We compare these findings to known results for simple random walks and partial sums of multiplicative functions, like $\mu(n)$, making new conjectures as appropriate.

23 The Chow Polynomial of the Non-crossing Partition Lattice

(ID = WZ02152033)

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Abstract

In 2024, Ferroni, Matherne, and Vecchi introduced the Chow polynomial $\underline{H}_P$ of a poset P, generalizing the Chow polynomial of a matroid. The partition lattice is the set of partitions of $\{1, 2, ..., n\}$ ordered by refinement. The Chow polynomial of the partition lattice is a well-studied and interesting invariant. We study the Chow polynomial of a closely related poset, the non-crossing partition lattice. The non-crossing partition lattice is a subposet of the partition lattice and is related to various structures including parking functions and braid groups.

A recent result of Stump describes the Chow polynomials of R-labeled posets. Using Stanley's parking function R-labeling of the non-crossing partition lattice, we derive an explicit expression for the Chow polynomial of the non-crossing partition lattice in terms of the number of parking functions with a fixed descent set. We study the evaluations $\underline{H}_{NC_n}(0), \underline{H}_{NC_n}(-1)$, and $\underline{H}_{NC_n}(1)$ of the Chow polynomial of the non-crossing partition lattice of n elements. We provide explicit expressions for $\underline{H}_{NC_n}(0)$ and $\underline{H}_{NC_n}(-1)$. We conjecture that $\underline{H}_{NC_n}(1) = (n-1)^{n-2}$ and verify this statement for $n \leq 11$. This conjecture is particularly striking as similar patterns are not present in the partition lattice.

24 Estimating Flows of Partial Differential Equations using Time-Series Analysis (ID = AM02152224)

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Abstract

Consider a class of nonlinear, time-dependent partial differential equations resulting from modeling high-dimensional dynamical systems, such as those defined by the Navier–Stokes equations. Let $u(\mathbf{x}, t)$ be the system state over a spatial subdomain $\Omega \subset \mathbb{R}^3$ and time interval $[0, T]$, governed by:

$$\frac{\partial u}{\partial t} = \mathcal{N}[u],$$

where $\mathcal{N}$ is a (possibly nonlinear) spatial differential operator.

We utilize a framework for modeling the evolution of approximate solutions for these PDEs that capture the spatiotemporal behavior of u as follows. We construct a finite set of orthonormal basis functions or structured non-negative components $\{\phi_i(\mathbf{x})\}_{i=1}^r$ through matrix decomposition techniques applied to a finite collection of spatially discretized solution states $\{u(\mathbf{x}_j, t_k)\}$ sampled at discrete times t_k. These techniques include Principal Component Analysis (PCA) and Nonnegative Matrix Factorization (NMF), subject to algebraic constraints preserving physical interpretability. Thus, we approximate u as:

$$u(\mathbf{x}, t) \approx \sum_{i=1}^{r} a_i(t)\phi_i(\mathbf{x}),$$

where $a_i(t)$ are time-dependent modal amplitudes.

We develop a method to utilize the time-dependent modal amplitudes to predict future spatiotemporal behavior of u. The results show that this analysis allows for an extension of u to a larger time domain using limited resources. This approach refines the complex dynamics into a simplified form that is more straightforward to analyze.

25 Algorithm for group law on tropical elliptic curves

(ID = LC02165150)

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Abstract

In 2004, Vigeland defined a group law on a tropical elliptic curve $E \subset \mathbb{TP}^2$. This construction required performing a certain geometric move sending a pair $(P, Q) \in E^2$ not lying on a common tropical line under stable intersection to a new *good* pair (P', Q') that does. We construct an explicit algorithm requiring a single move for the *symmetric honeycomb form* as in work by Chan and Sturmfels (2012). We then construct, for a general embedding, a best possible (minimizing number of moves) algorithm that allows us to send a general pair of points on E^2 to any other pair with linearly equivalent sum. We apply these results to determine when we can deduce the expected group structure of the n-torsion points E_n of E with incidences due to good pairs. In particular, we conclude that

$$G_n = \langle E_n \mid a + b + c = 0 \text{ if } \{a, b, c\} = \ell \cap_{st} E \text{ for some tropical line } \ell, \mathcal{O} = 0\rangle \cong \mathbb{Z}/n\mathbb{Z}$$

if and only if $n = 1, 2$ or $6 \mid n$.

26 Error Bounds on Laplacian Eigenvector Approximations

(ID = DS02165035)

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Yale University [Mentor: Smita Krishnaswamy]

Abstract

Motivated by the practical need to reduce the cost of eigendecomposition on large graphs, we study the conditions under which the graph Laplacian eigenvectors can be approximated through harmonic extension using only an induced subgraph. Every graph is characterized by a Laplacian matrix $L \in \mathbb{R}^{n \times n}$. For a partition of nodes into two sets, S nodes to eliminate, and B the complement set, we can partition the Laplacian L using block matrices for the interior and boundary vertices:

$$L = \begin{bmatrix} L_{SS} & L_{SB} \\ L_{BS} & L_{BB} \end{bmatrix}$$

The entries of the Schur complement matrix are directly related to effective resistances over the entire graph, which provide a measure of global connectivity between the subgraph vertices. Our approach focuses on identifying subgraph selection methods that best preserve the global spectral structure, enabling accurate recovery of the ground truth eigenvectors. Our method computes the Schur complement of the Laplacian with respect to S, $L_{BB} - L_{BS} L_{SS}^{-1} L_{SB}$, from whose spectrum the eigenvectors are harmonically extended as an approximation to the true eigenvectors on the whole graph. This work presents in detail how various initial subset selections will affect error bounds on the approximations to the full graph and we derive theoretical upper bounds on the deviation between the extended and true eigenvectors. We further establish connections to Nyström-style subset selection methods for low-rank matrix approximation, offering insight into how different selection choices impact eigenvector prediction quality and convergence guarantees. Our results aim to inform the design of subset selection strategies for efficient and accurate approximations of graph spectral information.

27 Obstructing Relative Trisection Destabilizations

(ID = LA02183443)

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Abstract

A *relative trisection* of a compact, smooth 4-manifold X with boundary is a decomposition of X into three simple, diffeomorphic pieces. Every such structure can be uniquely represented by three families of closed curves on a surface $(\Sigma, \alpha, \beta, \gamma)$. Interestingly, relative trisections induce an open book decomposition of ∂X, which is a decomposition of ∂X into infinitely many isotopic surfaces with a common boundary. Each decomposition comes with a diffeomorphism that dictates how to glue the surfaces together to recover ∂X; we call this diffeomorphism the *monodromy* of the open book.

In this talk, we will discuss how to obtain a trisected, 4-dimensional filling of any open book decomposition whose monodromy is represented by a composition of Dehn twists. In addition, we will use the lantern relation to obstruct certain trisected 4-manifolds inducing the same open book of the 3–sphere from being related through an operation known as *relative destabilization.* This is the first known example where the 3-dimensional structure (open book destabilization) does not directly correspond to the 4-manifold structure (relative destabilization). We conjecture that the daisy relation yields a family of further obstructions to this correspondence. This is based on ongoing joint work with Teddy Astor, Nickolas Castro, Drake McAdams, and Joanna Nelson.

28 Classification of Galois Groups of Prime Degree
(ID = KC02191716)

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Abstract

Given an irreducible and separable polynomial $f(x)$ of degree n over a field K, an effective way to compute the Galois group $\mathrm{Gal}(f/K) \subset S_n$ is by considering the factorization over K of various resolvent polynomials associated to $f(x)$. This method, introduced in the 18th century by Lagrange, used by Galois in the 19th century, and further developed by Stauduhar, Cohen, and others in the 1970s and 1980s, has been implemented in low degree in computer algebra systems such as `PARI/GP`, `Sage`, and `Magma`. My work is concerned with recognizing Galois groups of polynomials of prime degree p, with a view towards the inverse Galois problem over $\mathbb{Q}$.

First, I combine and harmonize various classification results for transitive subgroups of prime degree due to Galois, Feit, and which require the classification of finite simple groups. Second, generalizing results of Jensen, Ledet, Lui, and others, I determine the factorization of several resolvant polynomials for Galois groups of prime degree p, which enables a complete characterization of when $\mathrm{Gal}(f/K)$ is a given transitive subgroup of S_p. This characterization is easiest for primes p with a certain property, which we call *typical*. Aside from the primes $p = 11, 23$, where S_p contains sporadic simple groups, there is a purely number theoretic condition ensuring that p is typical, and results from analytic number theory show that most primes are typical. The atypical case is harder to analyze, and is the subject of various conjectures, though I can still make some progress.

The next step in this work is to use these criteria to realize new Galois groups of prime degree, as part of the inverse Galois problem over $\mathbb{Q}$.

29 Optimal Reinsurance under Utility-Based Preferences Risk Sharing (ID = SY03091606)

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The Ohio State University [Mentor: Linfeng Zhang]

Abstract

Classical risk-sharing models are often based on Pareto optimality, wherein the agents' preferences are characterized through their respective risk measures. However, a large class of risk measures, such as Tail Value-at-Risk (TVaR), are subadditive, which tend to yield extreme and unrealistic solutions concentrating (tail) risk entirely on one agent. Motivated by this limitation, our study introduces utility-based preferences to model the risk-sharing between two agents (e.g., insurer and reinsurer). We shall show that under appropriate assumptions, partial risk sharing can outperform full risk shifting. Our goal is to explore equilibrium conditions and structural implications of optimal risk allocation under this more general, preference-sensitive framework.

30 Using Positive Kernel Cone Extreme Ray Computation to Detect Minimal Directed Cycles

(ID = JF07123911)

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The Ohio State University [Mentor: Sanjeevi Krishnan]

Abstract

We present an alternative approach for detecting directed cycles in graphs by computing the intersection of the null space of the incidence matrix with the positive orthant. Instead of relying on classical DFS-based algorithms like Tarjan's, we identify extremal rays of the positive kernel cone via linear programming. Each extreme ray corresponds to a minimal directed cycle. This method provides a geometric and topological perspective, bridging graph theory with homological algebra. We evaluate our algorithm on multiple graph examples and compare its computational complexity to Tarjan's method. While our approach offers richer structural insight, it requires solving multiple LPs, making it slower in practice. Nonetheless, it opens new avenues for higher-dimensional cycle detection.

31 Math History Inspires Math Research: Napier's Calculating Machine Reimagined (ID = PK11170253)

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Abstract

The Scottish mathematician John Napier, primarily associated with the invention of logarithms in the 1600s, also developed several computational devices. In this research, we focus on his Chessboard Abacus and expand this concept to an algebraic extension of the base ϕ field where ϕ is the Golden Ratio. This expansion allows for the four primary arithmetic operations as well as square root approximations to be completed in base ϕ. Our work connects a geometric method to algebraic calculations using historical methods in a modern application.

32 Linear Recurrence Relations in Ribbon Quasai-Tree Subgraphs

(ID = KL18123602)

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The Ohio State University [Mentor: Sergei Chmutov]

Abstract

We consider certain sequences of ribbon graphs, topological multigraphs where vertices are disks and edges are ribbons connecting these disks. Existing research has found that if you count those spanning subgraphs of graphs in these sequences with exactly one connected boundary component, you can find the Fibonacci numbers, the Lucas numbers, or other sequences with the same recurrence relation.

We expand upon earlier methods used, consider more sequences, and generalize by counting subgraphs with exactly n boundary components. By these methods we find more complex linear recurrence relations, and we discover that counting subgraphs with $n + 1$ boundary components changes these relations in a predictable way.