Young Mathematicians Conference 2022

The Ohio State University, August 12-14

Distinct Angles and Angle Chains in Three Dimensions

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Sunday August 14, Session B, 11:30am-11:50am

Abstract of Report Talk: In 1946, Erdős posed the distinct distance problem, which seeks to find the minimum number of distinct distances between pairs of points selected from any configuration of n points in the plane. The problem has since been explored along with many variants, including ones that extend the problem into higher dimensions. Less studied but no less intriguing is Erdős distinct angle problem, which seeks to find point configurations in the plane in general position (no three points on a line and no four points on a circle) that minimize the number of distinct angles. In their recent paper "Distinct Angles in General Position", Fleischmann, Konyagin, Miller, Palsson, Pesikoff, and Wolf use a logarithmic spiral to establish an upper bound of $O(n^2)$ on A_{gen}, the minimum number of distinct angles, and of $O(\sqrt{n})$ on R_{gen}, the minimum possible maximum size of a subset of n points containing all distinct angles.

We consider the question of distinct angles in three dimensions, provide bounds on A_{gen} and R_{gen} in this setting, and explore the necessity of extending the definition of general position to prohibit four points on a plane or five on a sphere. We then discuss the difficulties of constructing a point configuration in general position with fewer than $O(n^2)$ distinct angles in any number of dimensions. We examine explicit constructions of point configurations in $\mathbb{R}^3$, such as the cylindrical helix and the conchospiral, which use self-similarity to minimize the number of distinct angles.

Finally, we apply these constructions to a variant of a question on angle chains considered by Palsson, Senger, and Wolf in their paper "Angle Chains and Pinned Variants", which establishes bounds for the number of subsets of $k+2$ points selected from a given set of points E and forming a specified k-tuple of angles. Our work extends these results by studying constructions of distinct angle chains and providing bounds on the number of such chains in higher dimensions.

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Revisiting the Upper Bound of the Number of Rectangulations of a Finite, Planar Point Set

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Baruch College [Mentor:Adam Sheffer]

Friday August 12, Session C, 2:45pm-3:05pm

Abstract of Report Talk: Triangulations of point sets have become ubiquitous in the research world. They are commonly used in graph theory, computational geometry, robot motion planning, computer graphics, and more. Rectangulations are a variant of triangulations, where we partition a region into axis-parallel rectangles. Over the past two decades, many works have studied rectangulations, leading to a rich theory.
We prove that every set of n points in the plane has at most $(16+\frac{5}{6})^n$ rectangulations. This improves upon the previous upper bound of 18^n, which has remained unchanged for 12 years. Our proof is based on the cross-graph charging-scheme technique.

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Density of Elliptic Dedekind Sums in Non-Euclidean Fields

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Sunday August 14, Session C, 11:30am-11:50am

Abstract of Report Talk: Elliptic Dedekind sums are a particular generalization of classical Dedekind sums, which arise in the disparate fields of number theory and topology. Although classical Dedekind sums were shown to be dense in the real plane by Dean Hickerson, density results for elliptic Dedekind sums have proven to be much more difficult. Using continued fractions, Hiroshi Ito established the density of the elliptic Dedekind sum in only the five Euclidean imaginary quadratic fields. This project aims to show the density of elliptic Dedekind sums in the infinitely many non-Euclidean imaginary quadratic fields by applying a recently established algorithm of Daniel E. Martin for complex continued fractions in these fields. In this talk, we present progress towards proving the density of elliptic Dedekind sums in non-Euclidean imaginary quadratic fields.

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Benfordness of lower-dimensional spaces resulting from hyper-box fragmentation

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Sunday August 14, Session A, 11:00am-11:20am

Abstract of Report Talk: One might expect that in a data set, each number from 1 to 9 is equally likely to be a leading digit and occurs roughly 11% of the time. However, this fails for numerous systems, where the probability of the first digit being d base 10 is about $\log_{10}(1 + 1/d)$, meaning the probability of a leading 1 is about 30%. This phenomenon was discovered by Simon Newcomb while looking at logarithm tables in the 1880s, when he noticed the pages corresponding to numbers with small logarithms were more worn, indicating more use. This was rediscovered by Frank Benford 50 years later in a variety of very distinct systems and is now known as Benford's law. Historically, Benford's law has been used by the IRS to detect potential cases of tax fraud, as well by academics to detect data fabrication. It also arises in many special mathematical sequences (the Fibonacci numbers) and in physics (nuclear fragmentation).

Recently, mathematicians and physicists have explored the relationship between Benford's Law and fragmentation processes. Consider the following: Suppose you have a stick of length L. Draw p_1 from a uniform probability distribution on $(0, 1)$. This fragments the stick into two parts of lengths Lp_1 and $L(1-p_1)$. On each substick, draw another independent probability (p_2 and p_3, respectively) from the same distribution. Repeat this fragmentation process n times. This process, and its generalization to the volumes of m-dimensional boxes, was proved to be Benford.

We continue this work by looking at d-dimensional subspaces of m-dimensional boxes under the generalized fragmentation process. Using techniques from Fourier and harmonic analysis, as well as approximation theory, we prove these objects converge to Benford behavior. Additionally, previous work required the probability distribution of the cuts to satisfy certain properties, which were known to hold only in special cases. We expand the set of known examples through delicate analysis of sums of products of Mellin transforms of density convolutions.

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Exploration of the Pocket Rubik's Group

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UMass Lowell [Mentor:Daniel Glasscock]

Poster Presentation

Abstract of Poster Presentation: An introduction to Group Theory will often include references to the Rubik's Group. The Rubik's Group has elements defined by letters representing moves performed on a Rubik's Cube, where sets of moves resulting in the same cube will be considered the same element. In-depth analysis is often not done, partly due to the cardinality of over 43 quintillion. A more manageable cardinality of just over 3 million is held by the Pocket Rubik's Group, which can be defined similarly to the Rubik's Group but with a 2x2x2 Cube. The group was generated using GAP, a system for computational discrete algebra. Utilizing the quarter-turn metric, treating a ? radian rotation as two turns, an adjacency matrix was produced, and the diameter of the Pocket Rubik's Group was confirmed to be 14. The group is defined by cyclic generators for programming efficiency. The group is also defined as a free group of 3 generators and a set of relations. Exploring the symmetry, the elements in the group can further be condensed. As associativity fails, the Pocket Rubik's Group fails to be a group under additional symmetry. Utilizing the symmetry, however, allows a simplified graph to be generated, which more succinctly presents the behavior of the group when compared to the Cayley Graph. The adjacency matrix can be transformed into a stochastic matrix, which will then be utilized for Markov chain analysis to understand better how quickly some mixing algorithms for the cube will disperse and answer the question by many cube enthusiasts: When is a cube properly mixed??

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Lower Volume Bounds for Families of Knotoids

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Saturday August 13, Session A, 11:00am-11:20am

Abstract of Report Talk: We present several new constructions which allow us to extend the notion of hyperbolicity to a generalization of knots called knotoids. We prove that a large family of knotoids, namely, prime, alternating knotoids of height 1, are all hyperbolic under one of these constructions. Since hyperbolic volumes are well-ordered, it makes sense to ask which knotoid attains the minimum volume over this family. The problem of finding minimum volume or low volume manifolds is fundamental to the field of hyperbolic geometry, and it remains an open problem for many classes of manifolds. Motivated by this problem, we subsequently prove some lower bounds on volume for certain subfamilies of prime, alternating, height 1 knotoids. For example, we are able to find the minimum volume rational knotoid by analyzing the corresponding horoball packings and surgery slopes. Extending this result to broader subfamilies would greatly increase our understanding of these particular knotoids and the manifolds they correspond to under our construction.

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Locality in the Sum-Rank Metric: Bounds, Optimality, and Constructions

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Sunday August 14, Session C, 12:00pm-12:20pm

Abstract of Report Talk: In coding theory, the concept of locality describes the number of erasures a code can correct and the amount of data that needs to be accessed to do so. Codes that leverage the relationship between large correction capability and a fast correction procedure have been extensively studied in two metrics, where data is encoded into a vector or matrix. We first detail our generalization of locality that holds for data encoded as tuples of matrices-itself an extension of the two previous metrics. Taking advantage of the properties this abstraction preserves, we then use the algebraic structure of field extensions, and combinatorial arguments, to prove multiple tighter bounds on minimum distance-a concept associated with error correction-with respect to different notions of in-matrix and cross-matrix locality. Lastly, we generalize and combine optimal codes in the previous metrics to construct new codes with varying notions of optimal erasure- and error-correction ability.

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Short-Range Differences of the Number of Summands of Zeckendorf Decompositions

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Williams College [Mentor:Steven Miller]

Sunday August 14, Session A, 12:00pm-12:20pm

Abstract of Report Talk: Zeckendorf's Theorem states that any non-negative integer m can be uniquely written as the sum of non-consecutive Fibonacci numbers $\{F_n\}$; this sum is called the *Zeckendorf decomposition* of m. It is natural to ask how many summands appear in the Zeckendorf decomposition of a given integer m, which is denoted by $Z(m)$. Lekkerkerker proved that the average value of $Z(X_n)$ where X_n is uniformly distributed in $[F_n, F_{n+1})$ is $\frac{1}{\phi+2}n + O(1)$. Kologlu, Kopp, Miller, and Wang expanded on Lekkerkerker's expectation result, showing that the normalized distribution of $Z(X_n)$ converges to a Gaussian as $n \to \infty$. In a similar spirit, Beckwith et al. investigated the statistic of gaps between the indices of summands in a Zeckendorf decomposition. They proved that the probability that the Zeckendorf decomposition of X_n has an index gap of length j exhibits a geometric decay with ratio ϕ^{-2}. These results hold not only for the Fibonacci sequence, but also for other, more general sequences known as *positive linear recurrence sequences*, or PLRS.

We continue the study of Zeckendorf and PLRS-based decompositions. One of the new statistics we investigate is the change in the number of summands of the Zeckendorf decomposition of m when we shift by $s > 0$, denoted by $\Delta_s(m) := Z(m+s) - Z(m)$. We interpret $\Delta_s(X_n)$ as a random variable where X_n is uniformly distributed in $[F_n, F_{n+1})$. With fixed s, we determine the limiting distribution of $\Delta_s(X_n)$ as $n \to \infty$. Among our findings, we prove that for any fixed s the limiting distribution of $\Delta_s(X_n)$ decays geometrically with ratio ϕ^{-2}. We also examine $s = s(n)$ a function of n satisfying $Z(s) \to \infty$ as $n \to \infty$. Such a limiting distribution of $\Delta_{s(n)}(X_n)$ exists and has a symmetric shape.

We also investigate the behavior of the random variable $T(X,Y) = Z(X) + Z(Y) - Z(X+Y)$. If $X, Y \in [F_n, F_{n+1})$ are iid uniformly distributed, then our moment computation suggests, and we are in the process of proving, that the statistic T is asymptotically normal. A key ingredient in understanding the distribution of T is the fluctuations of $Z(X)$ in a Fibonacci interval. Random sampling suggests that $Z(X)$ is rather stable with respect to the expectation computed by Lekkerkerker which results in a rare appearance of peaks and valleys in our sample. Motivated by this, we constructed a discrete measure with respect to $Z(X)$, and claim it should converge rapidly to the Lebesgue measure in a weak sense. With this claim as the stepping stone, we are able to carry out explicit moment computation, and further verify the Gaussianity observation.

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Random and Maximal Lengths of Zeckendorf Games

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Williams College [Mentor:Steven Miller]

Sunday August 14, Session A, 11:30am-11:50am

Abstract of Report Talk: Zeckendorf proved that any positive integer has a unique decomposition as a sum of non-consecutive Fibonacci numbers $F_1 = 1, F_2 = 2, F_{n+1} = F_n + F_{n-1}$. This inspired Miller to create a two person game played on a strip of boxes labeled in order with Fibonacci numbers $(1, 2, 3, 5, \dots)$. The game begins with n tokens deposited in the first box, and players alternate moves by either combining $(F_k \wedge F_{k+1} \mapsto F_{k+2})$ or splitting $(2F_k \mapsto F_{k-2} \ \wedge \ F_{k+1})$ tokens, which keeps the weighted (by what box it is in) sum of token values constant. The last player to move is the winner, which Baird-Smith et al. showed always occurs at the Zeckendorf decomposition of n.

The creation of this game led to many natural questions. Baird-Smith et al. provided a non-constructive proof that Player 2 has a winning strategy for all $n > 2$. They proved that the shortest game consists of $n - Z(n)$ moves, where $Z(n)$ is the number of summands in the Zeckendorf decomposition of n. Cusenza et al. computed the best-known upper bound for the length of the game using linear algebraic techniques, which is of order linear in n with constant equal to the golden mean squared, and discovered a strategy to obtain the longest game.

By adopting a combinatorial approach, we obtain multiple simpler proofs of bounds on game lengths. By utilizing this new perspective, we prove that, for all m between the lengths of the longest and shortest game, there exists a set of strategies leading to a game of length m. In ongoing work we expand on this by trying to prove that the distribution of lengths of all possible games for a fixed n tending to infinity converges to a Gaussian, as well as proving directly that if both players move randomly each has an equal chance of winning in the limit.

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Cell Packing in Epithilial Tissue

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NC State [Mentor:Sharon Lubkin]

Friday August 12, Session C, 3:45pm-4:05pm

Abstract of Summary Talk: To understand morphogenesis, the generation and alteration of biological form, knowing how cells pack and rearrange in tissues is important. Foam models can be used to study morphogenetic processes. Often, biologists use 2D "vertex models" for cell packing, where they are limited to a plane and the cell edges are straight. However, cells exist in three dimensions where they have curved edges and surfaces, so 2D vertex models have limitations. In this paper, we use 3D foam models with curved edges and surfaces to represent how cells pack inside of tissues. By using finite element methods, we find minimal energy configurations with volume and tension constraints. Using these results, we aim to create useful 3D models of epithelial tissue structure.

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Maximal skew sets of lines on a Hermitian surface and a modified Bron-Kerbosch algorithm

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University of Michigan [Mentor:Tim Ryan]

Friday August 12, Session B, 2:15pm-2:35pm

Abstract of Report Talk: At the heart of the geometry of a Hermitian surface X (a special surface which is defined over $\mathbb{F}_{p^{2e}}$ for some prime p and positive integer e) is studying its configuration of lines. In particular, we are interested in finding maximal skew sets of lines, and attempt to answer the open question on the possible sizes of maximal skew sets of lines on a Hermitian surface of degree d. First, we shed light on the geometry of Hermitian surfaces, and on the geometry of a larger class of surfaces called extremal surfaces. Our results also have interesting combinatorial consequences in the form of a classical incidence structure called a Hermitian generalized quadrangle. Finally, we note that counting maximal skew sets of lines on X is equivalent to counting maximal complete subgraphs (cliques) of a particular graph, so that our problem is equivalent to a special case of the clique problem. We give a new algorithm to solve a modified version of this problem; in particular, our algorithm will provide all sizes of maximal cliques.

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Giving Group Laws for Galois Gerbs

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Friday August 12, Session A, 2:15pm-2:35pm

Abstract of Report Talk: Given a finite Galois extension L/K, a Galois gerb (bound by $\mathbb{G}_m$) is a group extension of $\mathrm{Gal}(L/K)$ by $L^\times$. Using objects slightly different from $L^\times$, Kottwitz (2014) constructed three "global" Galois gerbs $\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3$ associated to an extension of global fields L/K. They encode an amazing amount of number-theoretic information; for example, $\mathcal{E}_1$ and $\mathcal{E}_2$ are constructed using the data of global class field theory and local class field theory, respectively.
It turns out we can study these global gerbs locally. When L_v/K_u is an extension of local fields, local class field theory is able to provide an abstract classification of Galois gerbs bound by $\mathbb{G}_m$. By providing a general framework to write down group laws for group extensions of abelian groups, we are able to describe these "local" Galois gerbs in certain cases. We will discuss what these group laws look like, how they are obtained, and provide examples.

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Regularity Conditions on Paths of the Fractional Gaussian Field on S^1 and the Torus

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Saturday August 13, Session A, 1:45pm-2:05pm

Abstract of Report Talk: In this poster, we begin by defining the Fractional Gaussian Field (FGF) X_s on S^1 with parameter s. For $s > \frac{1}{4}$ we index Gaussian random variables by $\theta \in [-\pi, \pi]$, and for $s \leq 1/4$ we index them by smooth, 2π-periodic functions. Using methods from Fourier analysis and probability, we prove that these definitions make sense (i.e that the series defining the FGF converges almost surely). We then prove that the paths of X_s are almost surely Lipschitz for $s > \frac{3}{4}$, $\frac{1}{k}$ Holder for $s > \frac{1}{2} + \frac{1}{2k}$, and k times differentiable for $s > \frac{1}{4} + \frac{k+1}{2}$.
The theory of FGF's on manifolds is currently a topic of interest to cosmologists studying cosmic microwave background radiation and particle physicists studying Liousville quantum gravity. To this end, we propose a definition for the FGF on the torus and explore similar path properties. Our current interest is in regularity conditions on the paths of the FGF on a general n-dimensional torus and the S^2.

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Spreading speed of Fisher-KPP equation and its application in the one-prey two-predator system

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Ohio State University [Mentor:King-Yeung Lam]

Friday August 12, Session C, 4:15pm-4:35pm

Abstract of Report Talk: We study the Fisher-Kolmogorov-Petrovskii-Piskunov (Fisher-KPP) equation $u_t - u_{xx} = u(r(x,t) - u)$ to understand the dynamics of population distribution changes in space, and are interested in investigating its spreading speed, which is useful in population dynamics and ecology. We first studied the simplified case, where $r(x,t)$ is a positive constant r_0, and verified that its spreading speed is $c_* = 2\sqrt{r_0}$. The heterogeneous version of the equation motivated by climate change, where $r(x,t) = g(x - c_1 t)$, is also considered. By using the tools like the generalized supersolutions and subsolutions, the detailed proof of the spreading speed in shifting environment with one or multiple shifts will be studied. This result will also be generalized to a biological system which models the spatial invasion of two predators feeding on a single prey species.

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Analytical and Computational Study of Darwinian Dynamics

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The Ohio State University [Mentor:Yulong Xing]

Saturday August 13, Session B, 11:30am-11:50am

Abstract of Report Talk: Natural selection and mutation play an important role while scientists study evolution. Given a model of a group of consumers with two resources, the consumers have a heritable trait that makes the usage of one resource more efficient than the other. Our research focuses on finding whether the consumer population will evolve towards a dominant type while assuming the mutation rate is small. The result will be presented in the form of a moving Dirac concentration that resembled the Darwinian tree using two different methods. The first method used is to utilize the Hamilton-Jacobi equation, which was conducted from a selection mutation equation. This Hamilton-Jacobi equation allows us to find the evolution of the trait in a monomorphic population and capture the branching event and further development trend after branching. The second method is to use the reaction-diffusion equation from the evolutionary game theory. It was analytically shown that the steady state concentrates (i) at a single location; (ii) at two locations simultaneously; or (iii) at two alternative locations. The transient dynamics toward the eventual outcome will be carefully analyzed through a combination of theoretical and numerical approaches.

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Rationality of Real Conic Bundles with Quartic Discriminant Curve

Mattie H Ji (matthew_ji@brown.edu)
University of Michigan [Mentor:Lena Ji]

Sunday August 14, Session C, 11:00am-11:20am

Abstract of Report Talk: A plane conic is a curve defined by a degree 2 polynomial, and a conic bundle is a morphism of smooth varieties $\pi : X \to S$ such that every fiber is a conic and its general member is smooth. In this talk, we take $S = \mathbb{P}^2$ and discuss the case when π is smooth away from a quartic curve $\Delta \subset \mathbb{P}^2$.
A beautiful theorem by Zeuthen classifies real smooth quartic curves into 6 topological types. We explore the relationship between the topology of Δ and X for a particular family of conic bundles, which were previously studied by Frei et al. This family admits additional geometric structure, which we exploit to classify the number of real connected components of X with the topological type of Δ. We use this topological result, together with the Magma and Sage computer algebra systems, to construct rational and irrational examples of conic bundles where Δ has different topological types.

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Generalizing Minimality Properties of Far-Difference Fibonacci Decompositions

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Williams College [Mentor:Steven Miller]

Saturday August 13, Session C, 11:00am-11:20am

Abstract of Report Talk: Zeckendorf proved that every positive integer can be decomposed uniquely as a sum of distinct, non-adjacent Fibonacci numbers, with the condition $F_1 = 1$, $F_2 = 2$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. This decomposition can be generalized to any positive linear recursion sequence (PLRS), which is a sequence with terms defined by the recurrence relation $H_n = c_1 H_{n-1} + \cdots + c_k H_{n-k}$ with $c_i \in \mathbf{Z}_{\geq 0}$. A paper by Cordwell et al. showed that these generalized Zeckendorf decompositions are summand minimal (in the sense that no other decomposition of a positive integer into a sum of the H_i uses fewer summands) if and only if the sequence $(c_1, \ldots, c_k)$ is weakly decreasing.

If we now consider Zeckendorf-style decompositions with coefficients either ± 1 (with additional rules to guarantee uniqueness), we arrive at what Hannah Alpert terms a "far-difference representation." Demontigny et al. showed that far-difference representations share many similarities with the Zeckendorf decomposition, including the uniqueness of decomposition when generalizing to PLRS's and the asymptotic Gaussianity for the number of summands within a Fibonacci interval. It is natural to expect other intrinsic properties of Zeckendorf decomposition to be successfully lifted to the far-difference case; among our results, we examine the similarities and differences between properties of these two decomposition forms.

Alpert proved the minimality of far-difference representations, and we generalize her results to extend to PLRS's, dealing with the combinatorial complications that arise from the signed decompositions. We do this by constructing a monovariant, generalizing the techniques in the Cordwell paper, to prove that the Far-Difference decomposition is summand minimal given an appropriate restriction on the coefficients defining the PLRS. As a further extension, we also explore criteria of a "Far-Difference Game" analogous to the Zeckendorf Game introduced and analyzed in 2018 by Baird-Smith et al.

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Sum and Difference Sets in Semidirect Products of Groups

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Saturday August 13, Session C, 11:30am-11:50am

Abstract of Report Talk:
Given a subset A of $\{0, 1, \ldots, N-1\}$, its sumset and difference set are given by

$$A + A = \{a_1 + a_2 : a_1, a_2 \in A\}$$
$$A - A = \{a_1 - a_2 : a_1, a_2 \in A\}.$$

Studying these sets is of fundamental importance in additive number theory: important problems such as the Goldbach and twin primes conjectures can be characterized in terms of the sum or difference sets of the prime numbers, while Fermat's last theorem can be stated in terms of the sumset of n^{th} powers. A problem of recent interest has been to understand the relative sizes of the sum and difference sets corresponding to a fixed set A. We intuitively expect that almost always $|A - A| > |A + A|$ because addition is commutative, but Martin and O'Bryant surprisingly proved that the proportion of the 2^N subsets A of $\{0, 1, \ldots, N-1\}$ which are sum-dominant, that is, that satisfy $|A - A| < |A + A|$, is bounded below by a positive value for all $N \geq 15$. They proved this by controlling the "fringe" elements of A, those close to 0 and $N - 1$, which have the most influence over whether elements are missing from the sum and difference sets.
More recently, several authors have examined analogous problems for groups other than the integers. The approach for finite groups is quite different due to the lack of fringes. Zhao proved asymptotics for numbers of sum-dominant subsets of finite abelian groups as the size of the group goes to infinity. Vissuet and Miller examined the problem for arbitrary finite groups G and proved that almost all subsets $A \subseteq G$ have $|A + A| = |A - A|$ as $|G| \to \infty$. For the dihedral group D_{2n}, they also conjectured that of the remaining sets, most are sum-dominant for any $n \geq 3$. Further progress on this conjecture was made by Haviland et al. in 2020.
We explore the dihedral group further by considering subsets with a fixed number of rotations and reflections. We then consider the broader setting of generalized dihedral groups and semidirect products of finite abelian groups.

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Combinatorial Formulas for the Equivariant Cohomology of Peterson Varieties

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George Mason University [Mentor:Rebecca Goldin]

Poster Session

Abstract of Poster Presentation: Our goal was to verify a conjecture about the decomposition of the restriction of Schubert classes associated with transpositions to the Peterson variety into a linear combination of Peterson classes. Using a corollary to Billey's formula, we reduced the conjecture to a more concise combinatorial question about counting reduced words for transpositions embedded into long words. We uncovered an elegant visual framework for understanding these combinatorial questions and proved our conjecture in a specific subcase. Future work will involve proving the remaining cases of the conjecture and extending our combinatorial strategy to as many types of Schubert classes as possible.

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Linear Preservers of Lorentz Spectra in Higher Dimensions

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UCSB [Mentor:Maribel Bueno]

Friday August 12, Session C, 1:45pm-2:05pm

Abstract of Report Talk:
For a given matrix $A \in M_n$ and a closed convex cone $K \subseteq \mathbb{R}^n$, the eigenvalue complementarity problem consists of finding $\lambda \in \mathbb{R}$ and $x \in K\backslash\{0\}$ such that $(A - \lambda I_n)x$ lies in the dual cone of K and $x^T(A - \lambda I_n)x = 0$. The special case where $K = \mathbb{R}^n$ amounts to the standard eigenvalue problem for A. This class of problems initially arose in systems with frictional contact in mechanics and has applications in physics, economics, and engineering.

In our work we consider the self-dual Lorentz cone $\mathcal{K}_n = \{(\xi, \eta) \mid \xi \in \mathbb{R}^{n-1}, \eta \in \mathbb{R}, \|\xi\| \le \eta\}$. We refer to the set $\sigma_L(A)$ of all solutions λ to the corresponding eigenvalue complementarity problem as the Lorentz spectrum of A. More specifically, our work concerns the characterization of the linear preservers of the Lorentz spectrum, that is, the linear maps $\phi : M_n \to M_n$ such that $\sigma_L(A) = \sigma_L(\phi(A))$ for all $A \in M_n$. For $n = 2$, it is known that ϕ is a linear preserver of the Lorentz spectrum if and only if it takes the form $A \mapsto PAP^{-1}$ where P is a 2×2 matrix of a particular structure. The goal of this project is to show that for $n \ge 3$, all linear preservers take the form

$$A \mapsto \begin{bmatrix} Q & \\ & 1 \end{bmatrix} A \begin{bmatrix} Q^T & \\ & 1 \end{bmatrix},$$

where $Q \in M_{n-1}$ is an orthogonal matrix. The converse is an established result. One feature that distinguishes the $n \ge 3$ case from the $n = 2$ case is that there exist $n \times n$ matrices with infinite Lorentz spectra for any $n \ge 3$. Our strategy to identify the Lorentz spectrum preservers involves, among other things, characterizing the $n \times n$ matrices with infinite Lorentz spectrum and finding ϕ-invariant subspaces of M_n, where ϕ is a Lorentz spectrum preserver.

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Spectral Analysis of Perturbed Complex Laplacians on the Rossi Sphere

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University of Michigan - Dearborn [Mentor:Yunus Zeytuncu]

Friday August 12, Session A, 4:15pm-4:35pm

Abstract of Report Talk: The Rossi sphere is a fundamental example of an abstract CR manifold that doesn't globally CR-embed in any $\mathbb{C}^N$. One can detect this by showing that the eigenvalues of the perturbed complex Laplacian $\Box_b^t$ get arbitrarily close to 0. In this project, we study the eigenvalues of this family of perturbed operators $\Box_b^t$ for $|t| < 1$. We look at the restrictions of these operators on the spaces of spherical harmonics. On these finite dimensional subspaces of $L^2(S^3)$ we analyze the matrix representations and after a careful set of manipulations we obtain eigenvalues. From this analysis, we establish spectral gaps as t approaches 0. We demonstrate that, as these gaps grow, the spectra of $\Box_b^t$ converge to the spectrum of the unperturbed operator $\Box_b$.

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An alternative method for calculating Bessel integrals appearing in L-function zero statisics

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Williams College [Mentor:Steven Miller]

Saturday August 13, Session B, 2:15pm-2:35pm

Abstract of Report Talk: In analytic number theory, L-functions allow one to work locally to build an object globally. Montgomery and Dyson discovered that there is a significant connection between the low-lying zeros of L-functions close to the central point $s = 1/2$ and the distribution of eigenvalues near 1 of specific random matrix ensembles.
In 2006, Hughes and Miller investigated the moments of a smooth counting function of the zeros near the central point of L-functions of weight k cuspidal newforms of prime level N. The agreement of their calculations with that from Random Matrix Theory provides evidence for the validity of the Katz-Sarnak conjectures, which propose a correspondence between the distribution of eigenvalues of random matrix ensembles and the density statistics of the zeros of L-function families. By changing variables they convert multi-dimensional integrals to one-dimensional integrals already considered in previous work, and at the cost of involved combinatorics show these agree with random matrix theory when the support of the Fourier transform of the test function is $1/(n-1)$. One expects such results to hold up to $2/n$, which was recently proved by Dell et al., but only after again having to analyze numerous delicate combinatorial sums.
Instead of converting the integrals and dealing with the non-trivial combinatorial obstructions, we instead directly analyze the multi-dimensional Bessel integrals to more easily prove agreement up to $2/n$, which is more amenable to attempts to further increase the support, which leads to better bounds on the order of vanishing of L-functions at the center of the critical strip, one of the most important problems in modern number theory (for example, one of the Clay Millennial Problems is precisely this for elliptic curves).

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Bounding ranks of cuspidal newforms through excised orthogonal ensembles

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Saturday August 13, Session B, 1:45pm-2:05pm

Abstract of Report Talk: A major program in modern number theory seeks to understand the surprising connections between the distributions of zeros of various families of L-functions and the distribution of eigenvalues of random matrices in classical compact groups. In a 2006 paper, Miller observed unexpected repulsion of zeros near the central point for elliptic curve L-functions that did not agree with standard random matrix theory ensembles. Dueñez, Huynh, Keating, Miller, and Snaith in 2011 created the excised orthogonal ensemble to explain these results, based on a result of Waldspurger and Kohnen-Zagier that the values of certain families of L-functions at the central point are discretized. Following the philosophy that the central value corresponds with the value at 1 of the characteristic polynomial, Dueñez et al. proposed placing additional restrictions on the random matrices that model the of these L-functions. After finding the correct cutoff value, their excised matrix ensemble successfully replicated the repulsion behavior of the family of elliptic-curve L-functions.
We generalize to a new excised matrix ensemble whose eigenvalue distribution agrees with the distribution of zeros of a broader family of L-functions, namely the L-functions arising from even Hecke eigenforms of weight k. For $k \in \{2, 4\}$ the methods of Dueñez et al. successfully translate; however, for $k > 4$ there are serious difficulties in controlling the asymptotic behavior in computing a viable cutoff value for the excised ensemble due to the convergence of the estimated number of functions that vanish at the central point. We discuss the new ideas needed to obtain an effective cutoff value for $k > 4$, thus completing the calculations for an excised matrix model for this family of L-functions. We observe that the cutoff value for our matrix ensemble goes to 0 as k increases, corresponding with the fact that the repulsion effect coming from discretization vanishes as the weights tend to infinity. We compute a large set of low lying zeros for families of Hecke eigenforms with different weights k and verify the decreasing behavior of the cutoff value and use these extensive numerical computations to confirm the theoretical predictions.

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Generalized harmonic estimates for the n-level density of L-functions

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Saturday August 13, Session B, 2:45pm-3:05pm

Abstract of Report Talk: Analytic Number Theory studies the global behavior of L-functions to gain deep arithmetic insights into the central objects in number theory. The distribution of the L-functions critical zeroes (those on the line $\mathrm{Re}(s) = 1/2$) controls many arithmetic structures, such as the distribution of primes and the number of points of an algebraic curve over a finite field. The behavior of an L-function at the central point $s = 1/2$ is related to many important problems; Birch and Swinnerton-Dyer conjectured that the multiplicity of the zero there corresponds to the rank of the group of rational points on the associated elliptic curve. For other problems, such as the class number, one needs to understand not just the behavior at the central point, but that of all nearby zeros.
Katz and Sarnak conjectured a correspondence between the n-level density statistics of zeros from families of L-functions with eigenvalues from random matrix ensembles. In a 2000 paper by Iwaniac, Luo, and Sarnak, they compute the 1-level density of families of L-functions and probe locations of zeros near the central point using a Schwartz test function ϕ whose Fourier transform is finitely supported. They show for certain families of L-functions that the sum over the zeros weighted by ϕ is equal to the integral of ϕ against a continuous measure $W(x)\,dx$ depending only on the symmetry of the family; this classification of L-functions comes from random matrix theory. If $\widehat{\phi}$ has arbitrarily large support, we could take ϕ approaching a delta spike and determine the behavior at the central point; thus the greater the support, the more we can deduce about the behavior at the key point of interest.
Iwaniac, Luo, and Sarnak showed agreement between the 1-level density and the integral provided that $\mathrm{supp}(\widehat{\phi}) \subset (-2, 2)$. Hypothesizing that a complex exponential sum involving primes exhibits square root cancellation, they extended the verifiable support to $(-22/9, 22/9)$. We determine an analog of the square root cancellation hypothesis for the 2-level density in order to increase the support beyond what is known, and use this to prove new world records for bounding the order of vanishing at the central point.

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Edge Determining Sets and Determining Index

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Hamilton College [Mentor:Sally Cockburn]

Saturday August 13, Session C, 4:15pm-4:35pm

Abstract of Summary Talk: A graph automorphism is a bijective mapping of the vertices that preserves adjacent vertices. A vertex determining set of a graph is a set of vertices such that the only automorphism that fixes those vertices is the identity. The size of a smallest such set is called the determining number, denoted Det(G). The determining number is a property of the graph capturing the degree of symmetry. We introduce the related concept of an edge determining set and determining index, Det'(G). We prove the relationship Det'(G) $\leq$ Det(G)$\leq$ 2 Det'(G) when Det(G)$\neq$ 1 and find the bounds to be sharp. Further, we investigate properties and tools of this new concept, as well as the determining index for several families of graphs.

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Learning Spheres, Chains in $\mathbb{F}_q^d$

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Williams College [Mentor:Alex Iosevich]

Sunday August 14, Session B, 12:00pm-12:20pm

Abstract of Report Talk: Consider a set X and a collection $\mathcal{H}$ of functions from X to $\{0,1\}$. We say that $\mathcal{H}$ shatters a finite set $C \subset X$ if the restriction of $\mathcal{H}$ to C yields all $2^{|C|}$ possible functions from C to $\{0,1\}$. The Vapnik-Chervonenkis (VC) dimension of $\mathcal{H}$ is the size of the largest set it shatters. VC-dimension plays a key role in computational learning theory: $\mathcal{H}$ is PAC (Probably Approximately Correct) learnable if and only if it has finite VC-dimension and has sample complexity bounded by linear functions of its VC-dimension.

Recently, two papers by Fitzpatrick et al. and Iosevich, McDonald, and Sun studied the VC-dimension of classes of functions $\mathcal{H}$ on subsets $E \subset \mathbb{F}_q^d$, where $\mathbb{F}_q^d$ is the d-dimensional vector space over the finite field $\mathbb{F}_q$. The first paper examined the class of indicator functions on spheres in $\mathbb{F}_q^2$, and the second studied indicator functions on hyperplanes in $\mathbb{F}_q^3$. Both show that when the subset E is sufficiently large, the VC-dimension of $\mathcal{H}$ is 3, the same as the entire vector space.

Following this work, we introduce the classifiers $\mathcal{H}_t^d = \{h_{u,v} : (u,v) \in \mathbb{F}_q^d \times \mathbb{F}_q^d\}$, where $h_{u,v}(x) = 1$ if $||x-u|| = ||x-v|| = t$ and 0 otherwise; this can be interpreted as a generalization of lower dimensional spheres, similar to the ones considered by Fitzpatrick et al. for $\mathbb{F}_q^2$. Using tools concerning the geometry of finite fields and by constructing explicit point configurations, we generalize our results for different classifiers in higher dimensional vector spaces over finite fields.

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Geometric Investigations of Mandelbrot Orbit Combinatorics

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Northern Michigan University [Mentor:Joshua Thompson]

Sunday August 14, Session B, 11:00am-11:20am

Abstract of Report Talk: The Mandelbrot set is now recently studied for its potential in the field of Combinatorics. In this study, I examine the Mandelbrot set and sequences of orbit lengths corresponding to bulbs along the cardioid boundary. Utilizing angled addresses to plot the orbits on $\mathbb{D}_r \subseteq \hat{\mathbb{C}}$, I discovered interesting alternative geometric perspectives of the Mandelbrot boundary. I also have preliminary data that I believe suggests the Julia set we know so well is but one perspective, and many more exist of which we are unaware.

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Geodesic Dynamics on Discrete Quotients of H2 x H2

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Saturday August 13, Session A, 4:15pm-4:35pm

Abstract of Report Talk: Dynamics of the geodesic flow on $\mathbb{H}^2/\Gamma$ where Γ is a Fuchsian (discrete) isometry group are well-understood. On $(\mathbb{H}^2 \times \mathbb{H}^2)/\Gamma$, substantially less is known. We examine recurrent geodesic trajectories on this manifold. The visual boundary of $\mathbb{H}^2 \times \mathbb{H}^2$ can be represented as the product $S^1 \times S^1 \times [0,1]$, where each S^1 is the visual boundary of $\mathbb{H}^2$ and the position in the interval $[0,1]$ represents the ratio between a trajectory's displacements in the first and second coordinates. It is known that, under this representation, the limits of geodesic trajectories recurrent in $(\mathbb{H}^2 \times \mathbb{H}^2)/\Gamma$ must fall within $S^1 \times S^1 \times \mathcal{L}_\Gamma$, where $\mathcal{L}_\Gamma \subseteq [0,1]$ is the projective representation of the limit cone of Γ. The limit cone is defined on a Γ containing only hyperbolic isometries as $\overline{\bigcup_{\gamma \in \Gamma} \mathbb{R}^+ \lambda(\gamma)}$, where λ represents the Jordan projection, and is known to be convex from a theorem of Y. Benoist. We completely characterize $\mathcal{L}_\Gamma$ for certain Γ isomorphic to the free group with 2 generators. To our knowledge, this is the first such example produced.

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Multivariate Fibonacci-Like Polynomials and Their Applications

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Saturday August 13, Session C, 1:45pm-2:05pm

Abstract of Report Talk: The Fibonacci polynomials are defined recursively as $f_n(x) = xf_{n-1}(x) + f_{n-2}(x)$, where $f_0(x) = 0$ and $f_1(x) = 1$. We generalize these polynomials to an arbitrary number of variables with the r-Bonacci polynomial:

$$F_n^{[r]}(x_1, x_2, ...x_r) = \begin{cases} 0 & 0 \leq n < r-1 \\ 1 & n = r-1 \\ \sum_{i=1}^{r} x_i F_{n-i}^{[r]} & n \geq r \end{cases}$$

We extend several well-known results such as an explicit Binet-like formula and a Cassini-like identity. Additionally, we prove that the terms and coefficients of the r-Bonacci polynomials generate integer partitions and use this to derive a connection to ordinary Bell polynomials and an explicit sum formula given by

$$F_n^{[r]} = \sum_{\substack{\alpha_1, \alpha_2, \ldots \alpha_r \geq 0 \\ \alpha_1 + 2\alpha_2 \cdots + r\alpha_r = n-r+1}} \binom{\alpha_1 + \cdots + \alpha_r}{\alpha_1, \ldots, \alpha_r} x_1^{\alpha_1} x_2^{\alpha_2} \cdots x_r^{\alpha_r}$$

Moreover, we derive identities that relate the r-Bonacci polynomials, exponential Bell polynomials, Fubini numbers, and the Stirling numbers of the second kind. Finally, we show that $F_n^{[r]}$ is irreducible over $\mathbb{C}$ for $n \geq r \geq 3$.

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Short Intervals Containing Squarefree Polynomials Over Finite Fields

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Towson University [Mentor:Nathan McNew]

Friday August 12, Session C, 2:15pm-2:35pm

Abstract of Report Talk: Much work has been done studying the distribution of squarefree integers in short intervals. Filaseta and Trifonov have shown that there exists a constant c such that the length $h(x) = cx^{\frac{1}{5}} \ln x$ guarantees the existence of a squarefree integer in the interval $(x, x + h(x)]$ for x sufficiently large.
There are many parallels between the integers and polynomials over finite fields $\mathbb{F}_q[x]$. In particular, they are both unique factorization domains. This fact allows us to study both squarefree integers and squarefree polynomials. The analogue of primes in $\mathbb{Z}$ is monic irreducible polynomials in $\mathbb{F}_q[x]$. Squarefree polynomials are polynomials that have no irreducible factors of multiplicity two or higher. Other parallels have previously been used to translate results in the integers to comparable results in $\mathbb{F}_q[x]$. Using established results concerning the distribution of squarefree integers in short intervals , we consider $\mathbb{F}_q[x]$ and study the size of intervals in which there exists a squarefree polynomial.
In this setting, we define an interval of length $h_q(f)$ centered at f to be

$$I\big(f, h_q(f)\big) = \left\{ g \in \mathbb{F}_q[x] \middle| \deg(g) = \deg(f),\ \deg(f-g) \leq h_q(f) \right\}.$$

We consider how to bound $h_q(f)$ such that we can guarantee there exists a squarefree polynomial in the interval. Previous work of Keating and Rudnick on polynomials over finite fields has shown that for any $h_q(f)$ such that $0 < h_q(f) < \deg(f) - 1$, one can find q large enough such that there exists a squarefree polynomial in the interval. In our work, we find bounds on $h_q(f)$ for fixed q instead that are comparable to the bound on the length $h(x) = cx^{\frac{1}{5}} \ln x$ for squarefree integers.

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Graph Theoretic Methods for Computing Quantum Invariants

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Saturday August 13, Session A, 11:30am-11:50am

Abstract of Report Talk: A knot is an embedding of S^1 into S^3. Distinguishing knots from one another (up to isotopy) is a fundamental problem in knot theory. One approach to such a problem is to assign knots an isotopy invariant polynomial. One famous knot polynomial was found in 1984 by Vaughan Jones, known as the Jones Polynomial. In the 1980s, when connections were found between quantum field theory and knots, these polynomials became interesting in their own right as quantum invariants. There are deep connections between knots, graphs, and these polynomials. In particular, we can derive a planar graph from a knot diagram, giving rise to computations of these polynomials via graphs instead. We present a computationally fast method for calculating quantum invariants for different classes of knots by exploiting this relationship between knots and graphs.

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The Brezis-Nirenberg problem for a system of divergence-form equations

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Towson University [Mentor:Mathew Gluck]

Saturday August 13, Session B, 11:00am-11:20am

Abstract of Report Talk: The Brezis-Nirenberg problem established both conditions that imply the existence and conditions that imply the nonexistence of positive solutions to a critically nonlinear partial differential equation. Our work is inspired by two separate extensions of the Brezis-Nirenberg problem. Alves et. al. discovered analogous conditions for a system generalization of the Brezis-Nirenberg problem, while Montenegro and de Moura considered a version of the Brezis-Nirenberg problem with a divergence form operator. We will discuss a problem obtained by combining these two extensions. In particular, we will provide an overview of the analogues of the conditions found in the inspiring works.

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Modeling the Effects of Media on COVID-19 Transmission

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Augusta University [Mentor:Eric Numfor]

Saturday August 13, Session B, 3:45pm-4:05pm

Abstract of Summary Talk: The severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) that emerged in Wuhan Province, China in December 2019 is a zoonotic disease, derived from viral particles in bats. In this project, we formulate an SEIR model of SARS-CoV-2 with two susceptible classes comprising individuals who are unconscious to COVID-19 and those who are conscious to the virus due to media coverage. The disease-free equilibrium of our model is derived, and the basic reproduction number is computed, using the next generation matrix approach. To identify parameters that are sensitive to the reproduction number, we studied the elasticity indices of the reproduction number with respect to each parameter and identified parameters that are most sensitive in increasing the reproduction number and those that are most sensitive in decreasing the reproduction number. Numerical simulations suggest that as more unconscious susceptible humans transition to conscious susceptible humans, there is a decrease in disease prevalence and a delay in the peak time of maximum prevalence in the population. Furthermore, an increase in the messaging rate of COVID-related information by conscious susceptible humans results in a decrease in the basic reproduction. The outcomes of our contour plots suggest the possibility of eradicating the virus from the population under different combinations of the messaging rate by conscious susceptible humans, the rate at which COVID-related information wanes and the transmission rate of the virus. Results of numerical simulations and contour plots highlight the importance of media in the transmission of SARS-CoV-2 in the population.

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Calculations in Tight Closure Theory

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Saturday August 13, Session C, 2:15pm-2:35pm

Abstract of Report Talk:
Tight closure has proven to be an invaluable tool for studying singularities in prime characteristic. In 2010, Brenner and Monsky gave a single, highly pathological example in which tight closure fails to commute with localization. However, the exact conditions under which tight closure fails to localize remain unknown. We replicate the methods of Brenner and Monsky to give a new, less singular example of a ring for which tight closure fails to localize. Our methods involve a surprising tiling argument using Sierpiński triangles which we believe is of independent interest.

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Analytical Methods for Weakly Nonlinear Oscillators and the Two-Timing Approach

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Poster Presentation

Abstract of Report Talk:
The harmonic oscillator is one of the most well-known differential equation driven systems in applied mathematics. While its physical existence is rather easy to understand, its mathematical solution can be more complicated. The differential equation that outlines the most basic version of this system is homogeneous, linear, and second order. It is commonly understood that the harmonic oscillator can be solved by traditional techniques - by using a characteristic equation or solving its corresponding linear system. An important subset of these ordinary differential equations are known as *weakly nonlinear oscillators* - systems that possess a nonlinear functional term, dampened by a very small non-zero constant, denoted ϵ. Through the addition of a nonlinear term, the study complicates - as an explicit solution no longer exists. For weakly damped oscillators, perturbation theory is typically applied - assuming the explicit solution has a power series expansion. Although the perturbation expansion produces an approximate solution, it possesses a *secular term* - a t term that is neither exponentially damped or periodic in behavior, breaking the harmonic nature of the system. To resolve this, a method known as *two-timing* is employed, by introducing a second time scale to soothe the secular term and eventually remove it. In this paper, a brief overview of harmonic oscillators is given, then two weakly nonlinear oscillators are introduced. For these two systems, perturbation theory is attempted to find an analytical solution, and then two-timing to remove the secular term. Finally, there will be an analysis of the damping term's effect on the accuracy of the two-timed solutions.

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A Primal Dual Method for Topological Changes in Optimal Adversarial Classification

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Saturday August 13, Session B, 4:15pm-4:35pm

Abstract of Report Talk: Recently proposed machine learning algorithms have utilized a hypothetical adversary during training to improve performance and robustness on classification. However, a rigorous understanding of the mechanisms during this performance remains incomplete. In this work, we study a non-parametric variational framework for classification in the presence of an adversary where the power of the adversary is defined by a positive ε. We describe an algorithm to construct an optimal classifier for any adversarial power. Prior work has shown that a set of uncoupled ordinary differential equations govern the evolution of the geometry of optimal adversarial classifiers in one dimension for small enough ε. We prove that optimal classifiers are governed by the same differential equations except for a finite number of instantaneous changes in topology and discontinuous movements in the endpoints of classification intervals. We rely on a novel primal dual method stemming from optimal transport theory to prove the optimality of our algorithm.

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Fast Computation of Generalized Dedekind Sums

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Saturday August 13, Session A, 2:15pm-2:35pm

Abstract of Report Talk: The classical Dedekind sum is well-studied both inside and outside of number theory due to its connections with the Dedekind eta function and its applications in topology and combinatorial geometry. There exists an algorithm to compute classical Dedekind sums in polynomial time, which has a close relation to the Euclidean algorithm. In 2020, Stucker, Vennos, and Young developed a generalized Dedekind sum defined on the congruence subgroup $\Gamma_0(N)$, which requires exponential time to compute from the definition. We construct an algorithm that reduces the complexity for computing generalized Dedekind sums from exponential to polynomial time. We do so by examining the structure of the congruence subgroups $\Gamma_0(N)$ and $\Gamma_1(N)$ in $\mathrm{SL}_2(\mathbb{Z})$, thereby treating it as a word-rewriting problem in group theory.

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Rigidity Properties of 4-Regular Graphs

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Budapest Semesters in Mathematics [Mentor:Jordan Tibor]

Saturday August 13, Session C, 3:45pm-4:05pm

Abstract of Report Talk: We present original results on strongly minimally 2-vertex globally rigid and 2-edge globally rigid graphs (edge-sparse graphs from which we can remove any vertex or edge, respectively, and obtain a rigid graph).
Rigidity, loosely at least, is what you'd expect intuitively. To begin to grasp the concept of rigidity, imagine a Tinker Toy construction (graph): does it bend under pressure (admit a continuous deformation?).
Setting the stage with a few original characterizations of 2-vertex globally rigid graphs in terms of connectedness, we proceed to provide a conjectured inductive construction for 2-vertex globally rigid graphs along with a class of beautiful, vertex-transitive (highly visually symmetric in the standard representation) graphs which we prove are 2-vertex globally rigid and relevant to our potential inductive construction.
Likewise for 2-edge globally rigid graphs, we prove a characterization in terms of connectedness along with an inductive construction, strengthening a result of some of the foremost researchers in rigidity.

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Spectral Analysis of the Complex Sub-Laplacian

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Friday August 12, Session A, 3:45pm-4:05pm

Abstract of Report Talk: In 1911, Hermann Weyl established a relationship between the asymptotic behavior of the Laplace operator's Dirichlet eigenvalues and the volume of the domain on which it acts. Since then, there have been many results – referred to as Weyl's Law – connecting the spectrum of differential operators to geometry, giving rise to the field now known as spectral geometry. Two canonical operators in the Cauchy-Riemann (CR) geometric setting are the complex Laplacian $\Box_b$ and the complex sub-Laplacian $\mathcal{L}$, defined by using complex sections in the tangent bundle. These two operators are the CR analogs of the Laplace-Beltrami operator and sub-Laplacians in the Riemannian setting. Inspired by earlier results for $\Box_b$, we prove an analog of Weyl's Law for $\mathcal{L}$ on $\mathbb{S}^{2n-1}$ using an explicit description of the spectrum of $\mathcal{L}$ and Karamata's Tauberian theorem. In particular, we express the leading coefficient in the asymptotic expansion of the eigenvalue counting function of $\mathcal{L}$ in terms of the volume of the sphere.
Continuing our analysis of $\mathcal{L}$, we estimate the Sobolev norms of the canonical solution operator of $\mathcal{L}$. We calculate the Sobolev norms by comparing the eigenvalues of the Laplace-Beltrami operator to those of $\mathcal{L}$. This calculation also reveals the best constant in these estimates. Additionally, we characterize the compactness of the solution operator of $\mathcal{L}$ by proving the convergence of its Schatten r-norm for particular values of r.

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Constructing Group Actions on Triads and Seventh Chords

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Friday August 12, Session B, 4:15pm-4:35pm

Abstract of Report Talk: Transformational theory, pioneered by David Lewin in 1987, uses group actions to analyze music. In this talk, we present the extension of simply transitive group actions to disjoint unions by constructing new musically motivated group actions. Our main example is a new group action on a set made up of major and minor triads and multiple types of seventh chords. We characterize the structure of our constructed groups as direct products or semi-direct products according to whether a certain bijection is equivariant or twisted equivariant. We also characterize Lewinnian dual groups (centralizers) in the new setting and revisit known groups from this perspective, such as the neo-Riemannian PLR-group. Musical examples are transformational analyses of a work by Rachmoninoff and jazz standards.

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Finding the Minimal Splitting Surface of the Ideal Regular Octahedron in the Poincaré Ball

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Yale University [Mentor:Franco Vargas-Pallete]

Friday August 12, Session B, 3:45pm-4:05pm

Abstract of Report Talk: The Poincaré Ball is the model of three dimensional hyperbolic space that holds the sphere at infinity. Using a discrete approach, we find the minimal splitting surface of the regular hyperbolic octahedron inscribed in the ball. In particular, we find the area-minimizing hypersurface that divides the positive and negative vertices of our octahedron. This problem is equivalent to finding a minimal surface of genus-2 with curvature $k \leq 1$ in a hyperbolic manifold due to the fact that the regular ideal octahedron tessellates the space for a group action. In this talk, we discuss our path to a solution: finding the discrete mean curvature flow (negative gradient flow for area) and applying this to a triangulation to continually minimize the area of our discrete hypersurface. We also discuss the computational challenges to this approach and implications to other discrete minimal area problems.

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Defining the Fractional Gaussian Field on the torus

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University of Connecticut [Mentor:Fabrice Baudoin]

Saturday August 13, Session A, 3:45pm-4:05pm

Abstract of Report Talk: In this talk we study random motion on compact manifolds, specifically Fractional Gaussian Fields (FGFs), via discrete approximations. These random processes are important for their relevance to research in early-universe cosmology and the Riemann Hypothesis. We define the discrete Laplacian on the n-torus ($\mathbb{T}^n$) and compute its eigenvectors and eigenvalues. These are used to define the discrete FGF (DFGF) and we extend the definition using linear interpolation to show convergence of the discrete FGF to the continuous FGF for $s > \frac{n}{4}$. Here s is a parameter determining the regularity of the continuous FGF. These convergence results are generalized to the log-correlated case by changing the domain of the DFGF. The FGF on the sphere is of current interest but much more difficult to define thus we propose two methods: using longitude and latitude points on the sphere and using triangulations to create a graph on the sphere. We discuss our progress as well as issues that we have encountered with these definitions.

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Cuspidal Projections of Products of Eisenstein Series

Amy E Woodall (amy.woodall713@gmail.com)
Clemson University [Mentor:Hui Xue]

Friday August 12, Session A, 2:45pm-3:05pm

Abstract of Report Talk: Modular forms are special kinds of holomorphic functions on the upper half-plane that have recently contributed to exciting results in the field of Number Theory due to their deep connections with Dirichlet series and elliptic curves. In this talk, we will give an introduction to modular forms and discuss products of Eisenstein series. We will show that the projection of a product of two or three Eisenstein series of level one onto the cuspidal subspace of modular forms is not an eigenvector for all of the Hecke operators unless the dimension of the cuspidal subspace is one. We have proved this result by using the Rankin-Selberg convolution method, showing explicit bounds, and checking finitely many cases computationally.

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On the Continued Fraction Expansion of Random Real Numbers

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Friday August 12, Session A, 1:45pm-2:05pm

Abstract of Report Talk: It is well-known that any irrational number x has a unique representation as an infinite continued fraction

$$x = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{\ddots}}} = [a_0; a_1, a_2, \dots],$$

where $a_0 = \lfloor x \rfloor$ and the "digits" $a_i = a_i(x)$, $i = 1, 2, 3, \dots$ are positive integers that are uniquely determined by x.
Gauss and Kuzmin showed that, for almost all real numbers x, the frequency of the digit k in the continued fraction expansion of x is given by $P(k) = \log_2 \left(1 + \frac{1}{k(k+2)}\right)$, where $\log_2$ denotes the base 2 logarithm. That is, almost all real numbers x satisfy

$$\lim_{n\to\infty} \frac{1}{n} \#\{1 \le i \le n : a_i(x) = k\} = P(k), \quad k = 1, 2, 3, \dots$$

We generalized this result in two directions: First, we considered the frequency with which the continued fraction digits of a random real number x belong to a given subset $A \subseteq \mathbb{N}$. For example, we showed that when A is the set of "shifted" primes $\{p - 1 : p \text{ prime}\}$, this frequency is $\log_2 (\pi^2/6)$. Secondly, we considered the frequencies of finite strings of digits in the continued fraction expansion of a random real number. For example, we showed that a string of k consecutive digits of 1 appears with frequency $|\log_2(1 + (-1)^k/F_{k+2}^2)|$, where F_n denotes the nth Fibonacci number.

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The Permutation Groups of Generalized Perfect Shuffles

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Friday August 12, Session B, 1:45pm-2:05pm

Abstract of Report Talk: A perfect shuffle splits a deck of $2n$ cards into two equal stacks and then interlaces the cards from the two stacks one after the other. There are two ways to do the perfect shuffle, referred to as an in shuffle and an out shuffle. In 1983, Diaconis, Graham, and Kantor determined the permutation group generated by in and out shuffles on a deck of $2n$ cards for all n. They concluded their work by asking whether similar results hold for so-called generalized perfect shuffles, where a deck of mn cards is split into m equal stacks. Motivated by their questions, we investigated the structure of the group generated by generalized perfect shuffles for a deck of m^k cards, together with m^y-shuffles, for all possible values of m, k, and y. We prove the group is isomorphic to the semi-direct product $\mathbb{Z}_2^a \rtimes \mathbb{Z}_b$, where a and b are completely determined by $k/\gcd(y,k)$ and the parity of $y/\gcd(y,k)$. In particular, the group structure is independent of the value of m.

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Explicit gaps between k-free integers

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Saturday August 13, Session A, 2:45pm-3:05pm

Abstract of Report Talk: When $k \geq 2$, a k-free integer is defined to be an integer which is not divisible by the kth power of of any prime. The Chinese Remainder Theorem can be used to construct arbitrarily long gaps in the sequence of k-free integers, however, these gaps are relatively small compared to the size of integers involved when they appear. Because of this, it becomes an interesting question to determine bounds for the length $h(x)$ of the interval $(x, x + h(x)]$ for which there exists a k-free integer in this interval. Trifonov and Filaseta have shown that in the square-free case when $k = 2$, there exists a constant c such that $h(x) = cx^{\frac{1}{5}} \ln x$ is admissible for x sufficiently large. Trifonov later generalized this result to show that in the k-free case, there exists a constant $c(k)$ such that $h(x) = c(k)x^{\frac{1}{2k+1}} \ln x$ for sufficiently large x. In our work we show that in the square-free case where $k = 2$, the constant of $c = 65$ suffices for $x \geq 2$ and the constant of $c = 5.8$ is admissible for sufficiently large x. We further show that in the case for cube-free integers where $k = 3$, the constant of $c = 225$ works for $x \geq 2$.

To find an explicit constant which is admissible for large x, we utilize the methods of Trifonov and Filaseta along with explicitly constructed polynomial identities and tools from approximation theory to bound the number of k-free integers in the interval of $(x, x + h(x)]$. These methods allow us to establish an explicit expression for admissible constants $c(k)$ which when optimized over the multiple parameters used provides an explicit bound on the number of k-free integers in the interval of $(x, x + h(x)]$. In the square-free and cube-free cases with $k = 2$ and $k = 3$, respectively, we are able to construct more precise bounds allowing us to optimize the constant further while also showing these constants are applicable for all $x \geq 2$.

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